Name _____

MATH 122B   Calculus and Analytic Geometry II   Fall 2005   Exam #1

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

   This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

   You may not use the symbolic computing features available on a calculator such as the TI-89.

1. For each of the following, give a definition equivalent to that used in the text or in class.

   (a) *definite integral of the integrable function f for the interval $[a, b]$.*   (6 points)

   (b) *antiderivative of the function f*   (6 points)

2. (a) State the First Fundamental Theorem of Calculus. Include the hypotheses and conclusion.   (6 points)

   (b) State the Second Fundamental Theorem of Calculus. Include the hypotheses and conclusion.   (6 points)

3. Explain the difference between the two statements   (6 points)

$$\int_a^b f(x)\, dx = \lim_{\Delta x \to 0} \sum_{k=1}^{n} f(x_k^*)\, \Delta x \qquad \text{and} \qquad \int_a^b f(x)\, dx = F(b) - F(a).$$

4. Use the definition of definite integral to compute the value of $\int_0^2 x^3\, dx$. In other words, compute the value of this definite integral as the limit of a Riemann sum (*not* using the First Fundamental Theorem of Calculus).   (10 points)

5. Approximate the value of $\int_1^2 \frac{\sin x}{x}\, dx$ using 5 terms and inputs of your own choice.   (10 points)

6. For each of the following, evaluate the given indefinite integral.   (8 points each)

   (a) $\int \left(x + \frac{1}{x} \right) dx$ (b) $\int \left(e^x + \frac{1}{x^2} \right) dx$

7. For each of the following, evaluate the given definite integral.   (10 points each)

   (a) $\int_1^3 \left(x^3 - \sqrt{x} \right) dx$ (b) $\int_{-2\pi}^{2\pi} \sin x\, dx$

8. Water flows into a lake through one stream at a rate given by $6t^2$ gallons per hour and out of the lake through another stream at a rate given by $4t$ gallons per hour. Determine the change in the amount of water in the lake during the interval from $t = 1$ to $t = 3$.   (10 points)

9. The plot below shows the graph of a function f. Let F be defined by $F(x) = \int_1^x f(t)\,dt$.

   (a) Estimate $F'(5)$.   (6 points)

   (b) Plot a graph of F on the axes given below. As part of this, determine a scale for the vertical axis and label the axis accordingly.   (6 points)