Extending the idea of definite integral

1. (a) Evaluate $\displaystyle\int_1^b \frac{1}{x^2}\,dx$ for $b > 1$.
   (b) Sketch a plot illustrating an area interpretation of your result from (a).
   (c) Evaluate the limit of your result from (a) as $b \to \infty$.
   (d) Sketch a plot illustrating an area interpretation of your result in (c).

2. (a) Evaluate $\displaystyle\int_1^b \frac{1}{x}\,dx$ for $b > 1$.
   (b) Sketch a plot illustrating an area interpretation of your result from (a).
   (c) Evaluate the limit of your result from (a) as $b \to \infty$.
   (d) Sketch a plot illustrating an area interpretation of your result in (c).

3. (a) Set up an inequality comparing x^2 and $x^2 + 1$ for $x > 1$.
   (b) Set up an inequality comparing $\dfrac{1}{x^2}$ and $\dfrac{1}{x^2+1}$ for $x > 1$.
   (c) Set up an inequality comparing $\displaystyle\int_1^b \frac{1}{x^2}\,dx$ and $\displaystyle\int_1^b \frac{1}{x^2+1}\,dx$.
   (d) Use your inequality from (c) and your result from Problem 1(c) to reach a conclusion about $\displaystyle\lim_{b\to\infty}\int_1^b \frac{1}{x^2+1}\,dx$. (Note: This conclusion will be in the form of an upper bound on the value of the limit rather than the exact value of the limit.)
   (e) Sketch a plot illustrating an area interpretation of your result in (d).

4. (a) Set up an inequality comparing x and $\sqrt{x}$ for $x > 1$.
   (b) Set up an inequality comparing $\dfrac{1}{x}$ and $\dfrac{1}{\sqrt{x}}$ for $x > 1$.
   (c) Set up an inequality comparing $\displaystyle\int_1^b \frac{1}{x}\,dx$ and $\displaystyle\int_1^b \frac{1}{\sqrt{x}}\,dx$.
   (d) Use your inequality from (c) and your result from Problem 2(c) to reach a conclusion about $\displaystyle\lim_{b\to\infty}\int_1^b \frac{1}{\sqrt{x}}\,dx$.
   (e) Sketch a plot illustrating an area interpretation of your result in (d).