Flow rate and accumulation

Consider a tank of water with a faucet and a drain. The volume of water in the tank can change at different rates depending on how the faucet and drain valves are set. There can be a net flow of water into the tank at some times and a net flow of water out of the tank at other times. Let t be time (measured in minutes) and $f(t)$ be the rate of change in the volume of water in the tank (measured in gallons per minute). Note that a positive value for $f(t)$ means that the volume of water in the tank is increasing and a negative value of $f(t)$ means that the volume of water in the tank is decreasing.

1. The table below gives measured rates for various times in a five-minute interval. Use this data to estimate how much water accumulates in the tank during this five minute interval.

t (min)	$f(t)$ (gal/min)
0.0	3.96
1.0	14.52
2.0	19.60
3.0	9.74
4.0	1.78
5.0	0.12

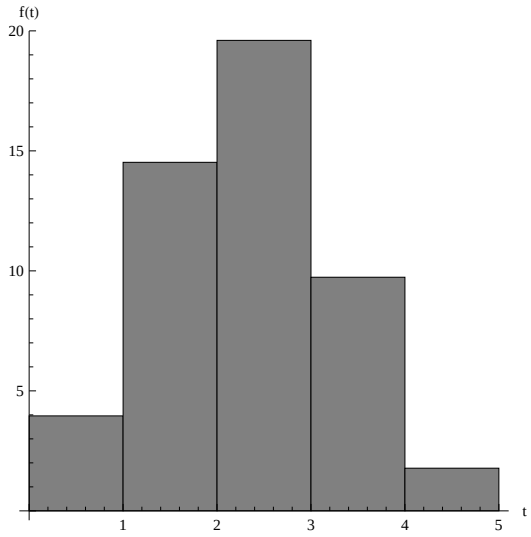
2. The table below gives more data for the measured rates during the same five-minute time interval as 1. Use this data to estimate how much water accumulates in the tank during this five minute interval.

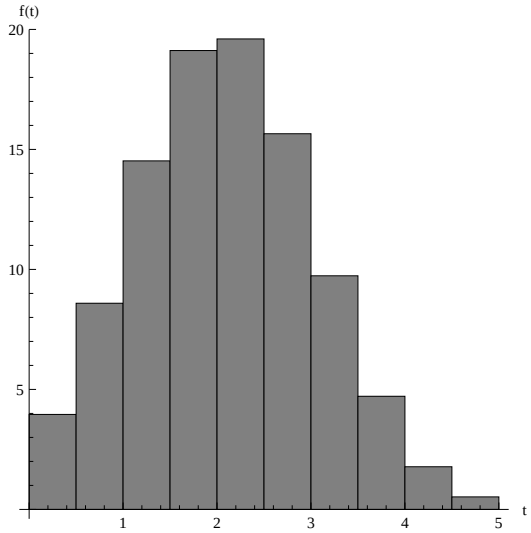
t (min)	$f(t)$ (gal/min)
0.0	3.96
0.5	8.60
1.0	14.52
1.5	19.12
2.0	19.60
2.5	15.65
3.0	9.74
3.5	4.71
4.0	1.78
4.5	0.52
5.0	0.12

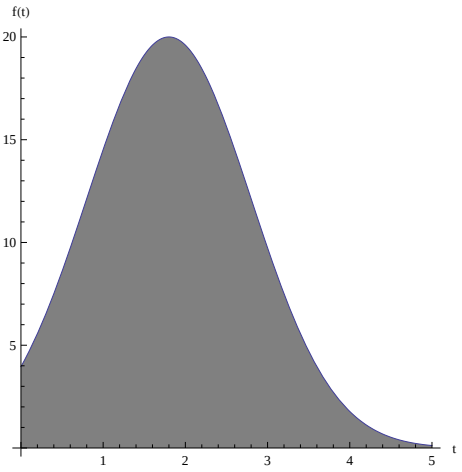
3. Now consider recording the flow rate $f(t)$ for all times between $t = 0$ and $t = 5$ minutes. We might get the data as a graph or as a formula (if the flow rate is controlled nicely). With information about the flow rate for all times in the interval, we should be able to compute the *exact* amount of water that accumulates in the tank during this five minute interval. Formulate a plan for this computation.

The plots below correspond to the three scenarios on the flip side.

Left endpoint estimates







Right endpoint estimates