Euler's method problems *with solutions*

Note: You might find it helpful to record your results in a table as you proceed through the calculations for each problem.

1. With a step size of $\Delta t = 0.2$, compute three steps of Euler's method to approximate the solution of $y' = -0.3y$ starting with $y = 25$ for $t = 1$.

   Solution:

   Calculations are shown in the following table.

Step	t	y	$\Delta y = -0.3y\Delta t$
0	1	25	$-0.3(25)(0.2) = -1.5$
1	$1 + 0.2 = 1.2$	$25 - 1.5 = 23.5$	$-0.3(23.5)(0.2) = -1.41$
2	$1.2 + 0.2 = 1.4$	$23.5 - 1.41 = 22.09$	$-0.3(22.09)(0.2) = -1.3254$
3	$1.2 + 0.2 = 1.6$	$22.09 - 1.3254 = 20.7646$	

   So, $y(1.6) \approx 20.76$.

2. With a step size of $\Delta x = 0.1$, compute three steps of Euler's method to approximate the solution of $y'(x) = e^{-x^2}$ starting with $y(0) = 0$.

   Solution:

   Calculations are shown in the following table.

Step	x	y	$\Delta y = e^{-x^2}\Delta x$
0	0	0	$e^{-0^2}(0.1) = 0.1$
1	$0 + 0.1 = 0.1$	$0 + 0.1 = 0.1$	$e^{-0.1^2}(0.1) = 0.099$
2	$0.1 + 0.1 = 0.2$	$0.1 + 0.099 = 0.199$	$e^{-0.2^2}(0.1) = 0.096$
3	$0.2 + 0.1 = 0.3$	$0.199 + 0.096 = 0.295$	

   So, $y(0.3) \approx 0.295$.

3. With a step size of $\Delta t = 0.4$, compute three steps of Euler's method to approximate the solution of $g'(t) = tg(t)$ starting with $g(0) = 5$.

   Solution:

   Calculations are shown in the following table.

Step	t	g	$\Delta g = tg\Delta t$
0	0	5	$(0)(5)(0.4) = 0$
1	$0 + 0.4 = 0.4$	$5 + 0 = 5$	$(0.4)(5)(0.4) = 0.8$
2	$0.4 + 0.4 = 0.8$	$5 + 0.8 = 5.8$	$(0.8)(5.8)(0.4) = 1.856$
3	$0.8 + 0.4 = 1.2$	$5.8 + 1.856 = 7.656$	

   So, $g(1.2) \approx 7.656$.

4. With a step size of $\Delta t = 0.5$, compute ten steps of Euler's method to approximate the solution of $R' = t - R$ starting with $R = 3$ for $t = 0$. Graph your computed points in a plot of R versus t.

Solution:

Calculations are shown in the following table.

Step	t	R	$\Delta R = (t - R)\Delta t$
0	0	3	$(0 - 3)(0.5) = -1.5$
1	$0 + 0.5 = 0.5$	$3 - 1.5 = 1.5$	$(0.5 - 1.5)(0.5) = -0.5$
2	$0.5 + 0.5 = 1.0$	$1.5 - 0.5 = 1.0$	$(1.0 - 1.0)(0.5) = 0$
3	$1.0 + 0.5 = 1.5$	$1.0 + 0 = 1.0$	$(1.5 - 1.0)(0.5) = 0.25$
4	$1.5 + 0.5 = 2.0$	$1.0 + 0.25 = 1.25$	$(2.0 - 1.25)(0.5) = 0.375$
5	$2.0 + 0.5 = 2.5$	$1.25 + 0.375 = 1.625$	$(2.5 - 1.625)(0.5) = 0.4375$
6	$2.5 + 0.5 = 3.0$	$1.625 + 0.4375 = 2.0625$	$(3.0 - 2.0625)(0.5) = 0.46875$
7	$3.0 + 0.5 = 3.5$	$2.0625 + 0.46875 = 2.53125$	$(3.5 - 2.53125)(0.5) = 0.484375$
8	$3.5 + 0.5 = 4.0$	$2.53125 + 0.484375 = 3.015625$	$(4.0 - 3.015625)(0.5) = 0.4921875$
9	$4.0 + 0.5 = 4.5$	$3.015625 + 0.4921875 = 3.5078125$	$(4.5 - 3.5078125)(0.5) = 0.49609375$
10	$4.5 + 0.5 = 5.0$	$3.5078125 + 0.49609375 = 4.00390625$	

A plot of these results is shown below.