More examples of constructing definite integrals

1. Charge is spread on a circle of radius R so that the density varies around the circle. (Note that circle here means the curve as opposed to a disk.) Let λ be the charge density measured in Coulombs per meter (C/m). Let θ measure the angle on the circle from a fixed reference ray (conventionally taken to be the positive x-axis). So, the charge density λ varies with angle θ.

   (a) Construct a definite integral to compute the total charge on the circle.

   (b) Compute the total charge if $\lambda(\theta) = \lambda_0(1 + \cos\theta)$ where λ_0 is a positive constant.

   (c) Get a numerical value for the total charge in (b) using the values $R = 0.25$ m and $\lambda_0 = 1.6 \times 10^{-3}$ C/m.

2. A hydrogen atom consists of one proton and one electron. A *free* hydrogen atom is one that experiences no external forces. In a free hydrogen atom, the electron can be in one of infinitely many discrete states. These states are labeled by three integers, usually denoted n, l, and m. For each state, there is an *electron location probability density* that gives the probability density (per volume) for the location of the electron as a function of position (measured with respect to the proton at the center of the atom). The states with $l = 0$ have probability densities that vary only with radial distance from the proton. (States with $l > 0$ have probability densities that also vary with angular directions.) Let r be the radial distance from the proton and let ρ be the electron probability density. So, probability density ρ varies with radial distance r.

   (a) Construct a definite integral to compute the total probability of finding an electron between radius $r = a$ and radius $r = b$.

   (b) The $n = 2$, $l = 0$, $m = 0$ state of a free hydrogen atom has an electron probability density given by

   $$\rho(r) = \frac{1}{32\pi}(2 - r)^2 e^{-r}.$$

   Here, the radial coordinate r is measured in units of *Bohr radii* where the Bohr radius is equal to about 5.3×10^{-11} meters. (So, for example, $r = 2$ means a radial distance of 2 Bohr radii or about 10.6×10^{-11} meters.)

   Compute the probability of an electron in the $n = 2$, $l = 0$, $m = 0$ state being between $r = 0$ and $r = 2$ Bohr radii.

   Note: For this and the following, it is sufficient to get a numerical estimate using technology.

   (c) Compute the probability of an electron in the $n = 2$, $l = 0$, $m = 0$ state being between $r = 2$ and $r = 4$ Bohr radii.

   (d) Compute the probability of an electron in the $n = 2$, $l = 0$, $m = 0$ state being between $r = 4$ and $r = 6$ Bohr radii.