Big ideas from Calculus I

1. Make a conjecture about $\lim_{x \to 0} \frac{\sin x}{x}$ and provide evidence to support your conjecture. (Note: For this problem, assume you do not yet know about derivatives and thus cannot use L'Hopital's rule.)

2. Evaluate the limit $\lim_{x \to 4} \frac{x-4}{x^2-16}$

3. Evaluate $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$.

4. State the definition of *derivative*.

5. Use the definition to compute the derivative of $f(x) = x^2$. (Hint: Refer to your work in Problem 3.)

6. For each of the following functions, give or compute the derivative.

   (a) $f(x) = 4x^3 + 5x^2 - 2x + 7$

   (b) $f(x) = \sin x$

   (c) $f(x) = \cos x$

   (d) $f(x) = e^x$

   (e) $f(t) = t^2 \sin t$

   (f) $g(x) = \cos(x^2)$

   (g) $w(y) = e^{\sin y}$

   (h) $h(x) = \sqrt{x^2 + 1}$

7. Determine the slope of the line tangent to the graph of $f(x) = 6x^3 - 2x$ for $x = 3$.

8. Determine all values of x for which the graph of $f(x) = 6x^3 - 2x$ has a horizontal tangent line.

9. The volume V (in gallons) of water in a tank is given as a function of time t (in hours) by $V = 100 + 20e^{-0.2t}$. Determine the rate at which water flows in or out of the tank for $t = 4$ hr.

10. Find all functions that have $f(x) = x^4 + 6x^3 + \sin x$ as derivative.

11. Find the antiderivative $F(x)$ of $f(x) = 12x^3$ that satisfies $F(1) = 5$.

12. An object moving along a line has a velocity of 3 m/s at $t = 0$ s. The object's acceleration for $t > 0$ is given by $a(t) = 6t$. Find the velocity function for this object.