Name ______________________________

MATH 181 Calculus and Analytic Geometry II Fall 2009 Exam #2

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

- *Each integral should be expressed entirely in terms of one integration variable.*
- *Pay attention to whether or not you are asked to evaluate integrals or just set them up.*
- *For this exam, you should not use symbolic features on your calculator if they are available on your model.*

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1. Set up an integral or integrals to compute the area of the region in the first quadrant bounded by the curves $y = x$, $y = 3x$, and $y = 1/x$. (15 points)

2. Consider a solid object that consists of
   - a cross section in the xy-plane that is a circle of radius 1 centered at the origin; and
   - square cross-sections perpendicular to the x-axis each having diagonal in the xy-plane extending from one side of the circle to the other.

   Set up and evaluate an integral or integrals to compute the volume of this solid. (18 points)

3. Consider the region in the first quadrant bounded by the curves $y = x^2$, $x + y = 6$, and $y = 1$. Set up an integral or integrals to compute the volume of the solid generated by rotating this region around the y-axis. (15 points)

4. (a) Set up an integral or integrals to compute the length of the ellipse given parametrically by $x = 4\cos\theta$ and $y = 3\sin\theta$. (12 points)

   (b) Find an "easy-to-compute" lower bound for the length of this curve. (3 points)

There is more on the flip side.

5. A rectangular piece of cloth is soaked in dye and then hung vertically to dry. As the cloth dries, the dye flows down so that more ends up at the bottom than at the top. The dried dye has a mass density that varies linearly from zero at the top edge to a maximum value at the bottom edge. Use H for the height of the cloth, W for the width of the cloth, and σ_0 for the maximum density.

   (a) Set up and evaluate an integral to compute the total mass of dye in the cloth. (12 points)

   (b) Explain why your result in (a) makes sense. (3 points)

6. Your friend is constructing an integral that involves slicing up a solid sphere of radius R into concentric spherical shells. Your friend correctly calculates

$$\Delta V = V_{\text{outer}} - V_{\text{inner}} = \frac{4}{3}\pi r_{\text{outer}}^3 - \frac{4}{3}\pi r_{\text{inner}}^3 = \frac{4}{3}\pi(r + \Delta r)^3 - \frac{4}{3}\pi r^3.$$

   After correctly expanding and simplifying, your friend arrives at

$$\Delta V = 4\pi r^2 \Delta r + 4\pi r(\Delta r)^2 + \frac{4}{3}\pi(\Delta r)^3.$$

   What should your friend use for dV in the integral being constructed? Explain why. (5 points)

7. Suppose we can measure how much calculus knowledge a person has. Let y be the amount of calculus knowledge in units called *smarts* and let t be time in hours. Suppose that as you study calculus, you gain knowledge according to

$$\frac{dy}{dt} = 0.4\sqrt{y}$$

   (a) State in words what this differential equation says about the rate of at which you gain calculus knowledge. (3 points)

   (b) Solve the differential equation to get a general solution. (9 points)

   (c) Find the specific solution for if you start with 25 smarts at time $t = 0$. (3 points)

   (d) How much knowledge will you have after 2 hours of studying if you start with 25 smarts? (2 points)