MATH 180   Calculus and Analytic Geometry I   Spring 2009   Exam #5

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

   This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.   (100 points total)

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1. For each of the following, state a definition equivalent that used in the text or class. Each definition should be self-contained.   (8 points each)

   (a) *F is an antiderivative of f*

   (b) *definite integral of f for* $[a, b]$

2. For each of the following, find all antiderivatives of the given function.   (8 points each)

   (a) $f(x) = 2x^3 + 5x^2 - 7x$ (b) $g(z) = \cos(5z) + e^{-3z}$

3. An object moves along a straight path with acceleration given by $a(t) = 3t$. At $t = 2$, the object is at position $s = 4$ with velocity $v = -1$. (Note: Units have been omitted for the sake of simplicity.)

   (a) Find the velocity function.   (6 points)

   (b) At what time does the object have a velocity of $v = 25$?   (3 points)

   (c) Find the position function.   (6 points)

4. Use the definition of definite integral to evaluate $\displaystyle\int_0^3 x^2 \, dx$.   (9 points)

5. State the Second Fundamental Theorem of Calculus. Your statement should be self-contained.
                                                                                         (10 points)

6. For each of the following, evaluate the given definite integral.   (10 points each)

   (a) $\displaystyle\int_{-1}^2 x^3 \, dx$ (b) $\displaystyle\int_1^3 \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

7. A child is having fun squeezing toothpaste out of a tube onto the floor. The toothpaste initially flows out at a fast rate but slows as the tube becomes harder to squeeze. The flow rate is modeled by $f(t) = 1 + \frac{1}{4}\cos(t)$ (in units of cubic centimeters per minute) for $t = 0$ to $t = \pi/2$.

   (a) Write down the definite integral that represents how much toothpaste accumulates on the floor during the given time interval.   (4 points)

   (b) Evaluate the integral from (a) to get the accumulation of toothpaste on the floor for the given time interval.   (6 points)

8. Your friend does the following calculation:

$$\int_{-1}^1 \frac{1}{x^2} \, dx = \int_{-1}^1 x^{-2} \, dx = -x^{-1}\Big|_{-1}^1 = -\frac{1}{x}\Big|_{-1}^1 = -\frac{1}{1} - \left(-\frac{1}{-1}\right) = -1 - 1 = -2$$

   Explain why we *cannot* conclude from this that the correct value of $\displaystyle\int_{-1}^1 \frac{1}{x^2} \, dx$ is -2.   (4 points)