Name _____

MATH 180 Calculus and Analytic Geometry I Spring 2009 Exam #4

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. Explain the distinction between a *local maximum* and a *global (or absolute) maximum.* (8 points)

2. Find the global minimum and the global maximum for $f(x) = \dfrac{x}{1+16x^2}$ on the interval $[0, 3]$. (12 points)

3. Consider the function $f(x) = xe^{-x/5}$ with domain consisting of all real numbers. Use calculus techniques for each of the following. Show enough detail for a reader to understand how you reach your conclusions.

   (a) Find all intervals of x for which the function is positive and all intervals of x for which the function is negative. (6 points)

   (b) Find all intervals of x for which the function is increasing and all intervals of x for which the function is decreasing. (6 points)

   (c) Find all intervals of x for which the function is concave up and all intervals of x for which the function is concave down. (6 points)

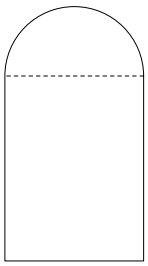
   (d) Analyze any limits that are relevant in understanding the graph of f. (4 points)

   (e) Sketch a graph of f and label any essential features (such as zeros, local extrema, and inflection points). Give x and y coordinates for each essential feature. (8 points)

4. Do *one* of the following two problems. Circle the problem number of the one you submit. (20 points)

   (A) You are choosing the dimensions of a rectangular window with the requirement that the area is 400 square inches. Your goal is to minimize the cost of the window frame. The material for the sides and the top of the window frame sells for 3 dollars per foot. The material for the bottom of the window frame sells for 5 dollars per foot. Find the dimensions that minimize the total cost.

   (B) You are designing a window in the shape of a rectangle capped by a semicircle as shown in the figure to the right. The perimeter is to be a total of 30 feet. Find the dimensions that maximize the area of the window. Note: Do not count the dashed line in the figure as part of the perimeter.



There is more on the flip side.

5. (a) State the Mean Value Theorem. (8 points)

   (b) On Monday, Deriba Merga (of Ethiopia) won the Boston Marathon with a time of 2.145 hours for the 26.2 mile race. Use the Mean Value Theorem to argue that the runner must have been moving at 12.21 miles per hour (rounded to the nearest one hundreth) for at least one instant of time during the race. Include a relevant plot to illustrate your use of the MVT. (8 points)

6. For each of the following, evaluate the given limit. (7 points each)

   (a) $\lim_{x \to 0} \dfrac{x \sin x}{1 - \cos x}$

   (b) $\lim_{x \to 1} x^{\frac{1}{1-x}}$