

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

- Consider the relation between x and y given by $6x + y^3 = x^2y$.
 - Show the the point $(x, y) = (4, 2)$ is on the graph for this relation. (3 points)
 - Determine the slope of the graph for this relation at the point $(x, y) = (4, 2)$. (8 points)
 - Sketch a plot that illustrates your results for (a) and (b).
Note: You do not need to show the graph of the relation. Just illustrate the specific information from (a) and (b). (3 points)
- Show how to determine the derivative of $\ln x$ using the fact that the derivative of e^x is e^x . (8 points)
- Suppose f is a function that has an inverse f^{-1} . Also, suppose $f(3) = 7$ and $f'(3) = 5$. Find the value of the derivative of f^{-1} that you can determine from the given information. (4 points)
- For each of the following, compute the derivative of the given function. (7 points each)
 - $f(x) = \ln(\sec x)$
 - $g(t) = t \ln t - t$
 - $y = \tan^{-1}(1 + x^2)$
 - $h(y) = y \sin^{-1} y$
- Geothermal activity is forcing some thick mud up through a hole onto a broad horizontal stretch of ground at a rate of 3000 cubic centimeters per minute. The mud spreads out in the shape of a cylindrical disk (centered on the hole) that always has a thickness of 5 centimeters. At what rate is the disk radius changing when the radius is 200 centimeters? (12 points)
- A person is walking at 3 feet per second on level ground away from a street light. The person is 6 feet tall and the street light is 15 feet above the ground. The person casts a shadow on the ground. How fast is the head end of the shadow moving on the ground? (12 points)
- The area A of a circle is related to the radius r by $A = \pi r^2$.
 - Find the corresponding relation between the differentials dA and dr . (6 points)
 - Find the corresponding relation between the percentage changes dA/A and dr/r . (4 points)
 - Suppose we measure the radius of a circle to within 1%. We then compute the area of the circle. What is the percentage error in the computed area? (2 points)
- With a step size of $\Delta t = 0.2$, compute three steps of Euler's method to approximate the solution of $f'(t) = f(t) - 2t$ starting with $f(1) = 3$. (10 points)