Name _____

MATH 180C Calculus and Analytic Geometry I Spring 2009 Exam #1

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. Consider the function $f(x) = \sqrt{9 - x^2}$.

   (a) Determine the domain and range of this function. (6 points)

   (b) In order to have an inverse function, the domain of this function must be restricted to something smaller than the domain from (a). Give a restricted domain for which $f(x) = \sqrt{9 - x^2}$ will have an inverse function. (4 points)

2. For this probem, do not use your calculator.

   (a) Make a plot of the function $f(x) = 5\sin(\pi x)$ showing at least one full cycle starting with $x = 0$. Include relevant scales on each axis. (5 points)

   (b) Make a plot of the function $f(x) = \tan x$ for x between $-3\pi/2$ and $3\pi/2$. Include relevant scales on each axis. (5 points)

3. Consider a function f with all real numbers as domain. The table to the right shows an infinite list of inputs x and the corresponding outputs $f(x)$. The number at the bottom of each list is the "end value" rather than an actual input or output.
   Based on this table, what can you conclude about $\lim_{x \to 5} f(x)$? Explain how you reach your conclusions. (8 points)

x	$f(x)$
4.9	3.1
4.99	3.01
4.999	3.001
4.9999	3.0001
4.99999	3.00001
⋮	⋮
5	3

4. For each of the following, evaluate the limit (or conclude "does not exist") using techniques that give an exact result if possible. Show enough details to make your methods clear to a reader. (9 points each)

   (a) $\lim_{x \to 5} \dfrac{x^2 - 25}{x - 5}$

   (b) $\lim_{p \to 9} \dfrac{p - 9}{\sqrt{p} - 3}$

   (c) $\lim_{z \to 0} \dfrac{\tan z}{z}$

   (d) $\lim_{x \to 5} \dfrac{x^2 + 25}{x + 5}$

There is more on the flip side.

5. Sketch the graph of a function f that has all of the following properties:

$$\lim_{x\to 2^-} f(x) = 5 \qquad \lim_{x\to 2^+} f(x) = 3 \qquad f(2) = 6$$

Note: You do *not* need to come up with a formula for the function you sketch. Just draw the graph of a function with all of these properties. (6 points)

6. The function $f(x) = \dfrac{4x}{x+3}$ is defined for all real numbers except $x = -3$.

   (a) Analyze the limits $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$. (4 points)

   (b) Analyze the limits $\lim_{x\to-3^-} f(x)$ and $\lim_{x\to-3^+} f(x)$. (4 points)

   (c) Sketch a plot that illustrates your results from (a) and (b). (4 points)

7. Evaluate $\lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ for the function $f(x) = x^2 + x$ and the value $x = 4$. (9 points)

8. Your friend, who once took calculus but did not understand it very well, says "To evaluate the limit of $f(x)$ at $x = a$, you can just plug a in for x." Write a brief paragraph explaining what is wrong with this statement. Include both a specific example and a general explanation of what limit means. (9 points)