Name ______________________________

MATH 122B   Calculus and Analytic Geometry II   Fall 2005   Exam #4

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

For problems requiring a conclusion about convergence or divergence, provide an argument or evidence to support your conclusion.

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1. What is the difference between a *sequence* and a *series*? (5 points)

2. State the definition of *the Taylor series of the function f based at x_0*. (5 points)

3. Use a direct or limit comparison argument to support a claim on whether the series $\sum_k^\infty \frac{|\sin(k)|}{k^{3/2}}$ converges or diverges. (15 points)

4. Use either the ratio test to determine if the series $\sum_k^\infty \frac{3^k (k!)^2}{(2k)!}$ converges or diverges. (15 points)

5. Determine if the series $\sum_k^\infty \frac{(-1)^k}{\sqrt{k^2+1}}$ converges absolutely, converges conditionally, or diverges. Give an argument to support your claim. (15 points)

6. Determine the interval of convergence for the power series $\sum_{k=0}^\infty \frac{(-1)^k}{k\,2^k} x^k$ (15 points)

7. Construct the Taylor series for the function $f(x) = xe^{x^2}$ based at $x = 0$. Give at least five nonzero terms. (15 points)

8. Using only arithmetic operations $(+, -, \times, \div)$ on your calculator, approximate $\cos(0.23)$ to within $\pm 10^{-8}$. (If you don't know how to do this, approximate $\cos(0.23)$ using an 8th degree Taylor polynomial for partial credit.) (15 points)

Useful facts

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x - x_0)^{n+1} \qquad \lim_{k\to\infty}\left(1 + \frac{1}{k}\right)^k = e$$