Course Objectives

For the final exam, a well-prepared student should be able to

- state a definition for each relevant term (e.g., antiderivative, definite integral,...) equivalent to that used in class or the text
- find antiderivatives of a given function using knowledge of derivative results and rules
- find antiderivitives of a given function using substitution
- read and use summation notation
- compute the exact value of a given simple definite integral using the definition of definite integral (as limit of a Riemann sum)
- estimate or bound the value of a given definite integral
- use properties of definite integrals to simplify or re-express a given expression involving definite integrals
- state the First Fundamental Theorem of Calculus
- use the First Fundamental Theorem of Calculus to compute the derivative of a function defined in terms of integration
- state the Second Fundamental Theorem of Calculus
- use the Second Fundamental Theorem of Calculus to compute the value of a given definite integral
- use properties of definite integrals for symmetric functions (even or odd) to simplify or re-express a given expression involving definite integrals
- set up and evaluate an appropriate definite integral to compute the accumulated change for a given rate of change and interval
- set up and evaluate an appropriate definite integral to compute the area (signed or total) of a given region bounded by the graph of a function and the horizontal axis
- distinguish among (1) the definition of definite integral, (2) the evaluation of definite integral given by the conclusion of FTC2, and (3) an interpretation of definite integral as accumulation or area
- in constructing a Riemann sum, identify the terms in an approximation for one piece that will make a non-zero contribution to the related definite integral
- construct and evaluate a definite integral to compute a quantity of interest, including
  - area of a planar region
  - volume of a solid region
  - length of a curve
  - total from a density
  - accumulation from a rate
- use reasonable judgement to decide whether to evaluate a definite integral exactly or to give a numerical estimate using technology
- find "easy-to-compute" lower and upper bounds as a consistency check on the result of computing a quantity using a definite integral
- use separation of variables to solve a given first-order differential equation
- use an initial condition to determine a specific value for the constant in a general solution to a first-order differential equation
- use a differential equation model to analyze a real-world phenomenon
- understand the connection between constant per capita (or percentage) rate of change and exponential change

- apply a variety of tools and strategies to the problem of finding an antiderivative for a given function, including
  - knowledge of basic derivative/antiderivative pairs
  - basic substitution
  - integration by parts
  - trigonometric substitution
  - re-expression of trigonometric functions using trigonometric identities
  - re-expression of a rational functions using division and partial fractions
- understand how simple methods of numerically approximating a definite integral can be combined to get better approximation methods
- understand how error, error bound, and tolerance are related for a numerical approximation of a definite integral
- compute a numerical approximation for a given definite integral that has error less than a given tolerance
- compute an Euler method approximation for the solution of a given first-order differential equation with a given initial condition
- use informal reasoning to make a conjecture and then build a rigorous argument to justify a final conclusion on whether a given thing (improper integral, sequence, series) converges or diverges
- determine whether a given improper integral converges or diverges, and if it converges, determine the value
- interpret an improper integral (for example, as area of an unbounded region or as accumulation over an unbounded interval of time)
- give informal definitions of *sequence*, *series*, and *power series*
- state basic rules and results for convergence of sequences
- determine whether a given sequences converges or diverges and, if it converges, determine the limit
- distinguish between the sequence of terms and the sequence of partial sums for a given series
- state basic rules and results for convergence of series (including p-series and geometric series)
- compute the sum of a given convergent geometric series
- state a comparison (direct or limit) argument to support a claim that a given series converges or diverges
- use the ratio test to analyze the convergence of a given series
- use the alternating series theorem to analyze the convergence of a given alternating series
- determine if a given series converges absolutely, converges conditionally, or diverges and give an argument to support the conclusion
- determine the interval of convergence for a given power series
- differentiate or integrate a given power series
- construct a Taylor polynomial or Taylor series for a given function based at a given point
- find a power series representation for a given function by directly computing a Taylor series or by relating to a known power series representation
- use a Taylor polynomial to approximate the value of a function for a given input
- use power series representations to deduce/prove facts about functions
- state Euler's formula and use Euler's formula in simple ways