Name ____________________

MATH 302 | Partial Differential Equations | Fall 2008 | Exam #2

Instructions: Do your own work. You may consult your class notes and the course text. Do not consult other sources. Do not discuss generalities or specifics of the exam with anyone except me.

Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work (both computations and reasoning).

The exam is due in class on Tuesday, October 28.

If you want to include animations from *Mathematica*, send me an e-mail with your *Mathematica* notebook as an attachment. Name the file you send `Math302Exam1_XX.nb` where XX are your initials and put "Math 302 Exam 2" in the subject line of the e-mail. Please use the options `AnimationRunning->False` for each animation.

---

1. Consider the following Cauchy problem for the heat equation

$$u_t = ku_{xx} \qquad \text{for } -\infty < x < \infty \text{ and } t > 0$$

$$u(x,0) = \begin{cases} 0 & \text{if } |x| > 1 \\ 1 & \text{if } -1 \le x < 0 \\ -1 & \text{if } 0 < x \le 1 \end{cases}$$

Find a nice formula for $u(x,t)$. Makes plots or an animation to illustrate the solution. (25 points)

2. For each of the following boundary situations, solve the heat equation $u_t = ku_{xx}$ for $x > 0$ and $t > 0$ with initial condition given by a unit point source at $x = a$. Makes plots or an animation to illustrate each solution.

   (a) the end at $x = 0$ is held at temperature 0

   (b) the end at $x = 0$ is perfectly insulated

(30 points)

3. Use a Laplace transform approach to solve the following:

$$\begin{aligned} u_t &= ku_{xx} && \text{for } x > 0,\ t > 0 \\ u_x(0,t) &= g(t) && \text{for } t > 0 \\ u(x,0) &= 0 && \text{for } x > 0 \end{aligned}$$

(25 points)

4. Based on what we have looked at in this course, argue that $H'(x) = \delta(x)$. (10 points)

5. For each of the heat equation and the wave equation, write a paragraph or two describing the main features of its solutions. (10 points)