Name ______________________________

MATH 302 Partial Differential Equations Fall 2008 Exam #1

Instructions: Do your own work. You may consult your class notes and the course text. Do not consult other sources. Do not discuss generalities or specifics of the exam with anyone except me.

Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work (both computations and reasoning).

The exam is due in class on Monday, September 29.

If you want to include animations from *Mathematica*, send me an e-mail with your *Mathematica* notebook as an attachment. Name the file you send `Math302Exam1_XX.nb` where XX are your initials and put "Math 302 Exam 1" in the subject line of the e-mail.

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1. Show that $u(x, y) = \ln(\sqrt{x^2 + y^2})$ satisfies Laplace's equation for all $(x, y) \neq (0, 0)$. (10 points)

2. Advection with constant speed $c > 0$ is modeled by the equation $u_t + cu_x = 0$ for $-\infty < x < \infty$ and $t > 0$. (26 points)
   (a) Consider a situation in which we specify an initial condition of the form $u(x, 0) = f(x)$ for $-\infty < x < \infty$. Find the specific solution for this situation. Does this solution determine a unique value of $u(x, t)$ for $-\infty < x < \infty$ and $t > 0$?
   (b) Consider a situation in which we specify a boundary condition of the form $u(0, t) = g(t)$ for $t > 0$. Find the specific solution for this situation. Does this solution determine a unique value of $u(x, t)$ for $-\infty < x < \infty$ and $t > 0$?
   (c) Consider a situation in which we independently specify both an initial condition of the form $u(x, 0) = f(x)$ for $-\infty < x < \infty$ and a boundary condition of the form $u(0, t) = g(t)$ for $t > 0$. Explore existence and uniqueness of a specific solution.

3. Consider the PDE $u_t + 3t^2 u_x = -4u$. (26 points)
   (a) Use the characteristic coordinates $\xi = x - t^3$ and $\tau = t$ to find the general solution for this PDE.
   (b) Find the specific solution of this PDE that satisfies $u(x, 0) = e^{-x^2}$.
   (c) Illustrate your specific solution from (c).
   (d) Thinking of u as the density of dye in a fluid flowing down a tube, describe the general situation modeled by this PDE and the specific situation given by your solution from (c).

4. Lava sometimes flows in a *lava tube*. Heat in the molten lava will both diffuse and advect (by being carried along with the flow). The rock walls of the tube act as essentially perfect thermal insulation. Consider modeling the temperature distribution in a lava tube that starts at a reservoir of lava and ends at the ocean. (26 points)
   (a) Develop a simple PDE model for the temperature in the lava tube as a function of position and time. As part of this, you will need to make assumptions about the physical situation. List the assumptions you make. Develop your model in terms of parameters rather than specific values. For example, label the length of the tube with something like L rather than giving a specific value such as 1 mile.
   (b) Find the steady-state solution to your model in (a) and plot your solution.

5. Write a paragraph or two explaining how analysis of PDEs relates to or compares with analysis of ODEs. You might comment on similarities, differences, and connections. Base this on your experience with MATH 301 (or its equivalent) and your experience so far in this course. (12 points)