

Laplace transform problems

1. Use a Laplace transform approach to solve the IVP $u'' + 4u = 4t + 8$ with $u(0) = 4$ and $u'(0) = -1$.
2. Use a Laplace transform approach to solve the IVP $u'' + 3u' + 2u = 6e^{-t}$ with $u(0) = 1$ and $u'(0) = 2$.
3. The *Heaviside step function* (also known as the *Heaviside theta function*) is defined as

$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t > 0 \end{cases}$$

Compute $\mathcal{L}[H(t - a)]$ where a is a positive constant.

4. Come up with a formula for $\mathcal{L}[u^{(n)}(t)]$ where $u^{(n)}$ is the n th derivative of u . You can do this by looking for a pattern for small values of n . However you come up with the formula, you should then use induction to *prove* that the formula holds for all natural numbers n .