

Toward a precise definition of limit

Warm-up 2: Hitting a target

1. Consider the function $f(x) = 4x$ for x near $a = 3$.
 - (a) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 1 of 12. That is, $f(x)$ must be between 11 and 13.
 - (b) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 0.5 of 12.
 - (c) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 0.2 of 12.
 - (d) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within ε of 12. Note: In this part, the “target radius” is a variable. You should expect your “launch pad radius” to depend on ε .

2. Consider the function $f(x) = x^2$ for x near $a = 3$.
 - (a) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 1 of 9. That is, $f(x)$ must be between 8 and 10.
 - (b) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 0.5 of 9.
 - (c) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within 0.2 of 9.
 - (d) Find all inputs x near $a = 3$ such that the outputs $f(x)$ are within ε of 9.