Name _____

MATH 121B Calculus and Analytic Geometry I Fall 2004 Exam #4

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

---

1. (a) State the Mean Value Theorem. (8 points)

   (b) Sketch and label a plot illustrating the conclusion of the Mean Value Theorem. (6 points)

2. For each of the following, evaluate the given limit. (6 points each)

   (a) $\lim_{x\to\infty} \dfrac{3x^4}{8x^4+x^2}$ (b) $\lim_{x\to\infty} \dfrac{x^2}{e^x}$ (c) $\lim_{x\to 0}(1-100x)^{\frac{1}{x}}$

3. Consider the function $f(x) = \dfrac{x^2}{x-4}$. Use calculus techniques for each of the following. Show enough detail for a reader to understand how you reach your conclusions.

   (a) Find and analyze any vertical asymptotes. (4 points)

   (b) Analyze any horizontal asymptotes. (4 points)

   (c) Find all intervals of inputs x for which the function is positive and the all intervals of inputs x for which the function is negative. (6 points)

   (d) Find all intervals of inputs x for which the function is increasing and all intervals of inputs x for which the function is decreasing. (6 points)

   (e) Find all intervals of inputs x for which the function is concave up and all intervals of inputs x for which the function is concave down. (6 points)

   (f) Sketch a graph of the function and label any asymptotes, zeros, local minima, local maxima, and inflection points. (8 points)

4. Consider the function $f(x) = x^3 - Ax^2$ where A is a positive constant. Use calculus techniques to do the following. (7 points each)

   (a) Show that the function has a local minimum at $x = \frac{2}{3}A$.

   (b) Show that the function has an inflection point at $x = \frac{1}{3}A$.

5. Consider an isosocles triangle with one side of length 6 and two sides of length 12. Look at a rectangle inscribed in this triangle with one edge of the rectangle on the short side of the triangle. Find the dimensions of the inscribed rectangle with the largest possible area. (20 points)