Name ________________________________

MATH 121B — Calculus and Analytic Geometry I — Fall 2004 — Exam #3

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. (a) Give the precise definition of the phrase L *is the limit of the function* f *at* a. (6 points)

   (b) Sketch a plot showing a geometric interpretation of the pieces in the precise definition of limit. (4 points)

   (c) Prove the limit statement $\lim_{x\to 5}(17-3x) = 2$ using the precise definition of limit. (6 points)

2. Consider the relation $x^3y - xy^2 = 60$

   (a) Compute $\dfrac{dy}{dx}$. (12 points)

   (b) Find the equation of the tangent line at the point $(4, 1)$. (6 points)

3. An inverted cone (i.e., pointy end down) is being filled with water at a rate of 2 cubic inches per minute. The cone is 15 inches high and has a radius of 5 inches. Find the rate at which the depth of the water is changing when the depth is 4 inches. Hint: The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$. (12 points)

4. A spot of light from a searchlight is sweeping horizontally along a straight wall (in search of several calculus students who have broken out of Thompson Hall). The searchlight is located 30 meters from the wall directly in front of a door. The searchlight is rotated on its mount at a rate of 0.5 radians per second. How fast is the spot of light moving along the wall at the instant the spot passes over the door? Hint: Draw a picture of the situation as seen from directly overhead. (12 points)

5. For constant temperature, the volume V and pressure P of an ideal gas are related by $V = \frac{c}{P}$ where c is a constant. Find an approximate relation between a percentage change in volume and a percentage change in pressure. (12 points)

6. Consider the function $y = x^2 + x$.

   (a) Compute the exact rise Δy for a run of Δx. (4 points)

   (b) Compute the approximate rise dy for a run of dx. (4 points)

   (c) Explain how dy is related to Δy for this specific example. (2 points)

7. Consider the function $f(x) = x^4 - 8x^2$.

   (a) Find all critical numbers for this function. (10 points)

   (b) Classify each critical number as a local minimizer, a local maximizer, or neither. (10 points)