

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

1. (a) Give a definition of *the derivative of the function* f (assuming f is differentiable). (6 points)
(b) Sketch a plot showing a geometric interpretation of the pieces in your definition of derivative. (6 points)
2. Compute the derivative of $f(x) = 5x^2$ using the definition of derivative. Do not use the results and rules of differentiation. (8 points)
3. The constant multiple rule for derivatives is $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$. Prove this starting from the definition of derivative. (6 points)
4. For each of the following, compute the derivative of the given function. Do reasonable simplification of the result. (8 points each)
 - (a) $f(x) = 3x^5 - 2x^2 + 4$
 - (b) $f(x) = \frac{x^2 - 4x}{x + 1}$
 - (c) $g(x) = x \cos(x^2)$
 - (d) $f(x) = x\sqrt{x^2 + 1}$
 - (e) $f(t) = \ln(\sin t)$
 - (f) $f(x) = e^{ax} \cos(bx)$ for constants a, b
5. Find the equation of the line tangent to the graph of $f(x) = -2x^3$ for $x = 2$. (8 points)
6. The *ideal gas law* relates the pressure P , volume V , and temperature T of a sample of gas. The relation is
$$PV = kT$$
where k is a constant. Compute the rate of change in pressure with respect to volume for constant temperature. (8 points)

7. The plot below shows the graph of a function f . Plot a graph of the derivative f' on the axis provided below. As part of this, determine a scale for the vertical axis and label the axis accordingly. (10 points)

