Mean of the Two-Dice Sum Data

Earlier this week, three groups collected 300 values for the sum on a pair of dice. Here is the data distribution with values arranged in increasing order:
2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 . . . 11 11 11 11 11 11 11 11 11 11 12 12 12 12 12 12 12 12 12
OK, I left out a bunch of values in the middle, but you get the idea. Each possible outcome (2 through 12) is repeated many times. We can record a count for each outcome and a proportion for each outcome. Here's those numbers:

Outcome	Count	Proportion
2	9	$\frac{9}{300}$
3	13	$\frac{13}{300}$
4	22	$\frac{22}{300}$
5	33	$\frac{33}{300}$
6	35	$\frac{35}{300}$
7	57	$\frac{57}{300}$
8	46	$\frac{46}{300}$
9	37	$\frac{37}{300}$
10	29	$\frac{29}{300}$
11	10	$\frac{10}{300}$
12	9	$\frac{9}{300}$
Total	300	$\frac{300}{300} = 1$

Now let's examine how to compute the mean of this distribution. To do this, we go back to the listing of all the values in the distribution and compute

$$\bar{x} = \frac{2 + 2 + 2 + 2 + \ldots + 12 + 12 + 12 + 12}{300}$$

The numerator is a sum of all 300 values. We can reorganize the arithmetic to get a different way of thinking about the mean. Using the counts from the table, we can replace repeated addition of the same value with multiplication and write this as

$$\bar{x} = \frac{(2 \times 9) + (3 \times 13) + (4 \times 22) + (5 \times 33) + \cdots + (10 \times 29) + (11 \times 10) + (12 \times 9)}{300}$$

Using rules for adding fractions, we can rewrite this as

$$\bar{x} = \frac{2 \times 9}{300} + \frac{3 \times 13}{300} + \frac{4 \times 22}{300} + \frac{5 \times 33}{300} + \cdots + \frac{10 \times 29}{300} + \frac{11 \times 10}{300} + \frac{12 \times 9}{300}$$

Finally, we can reorganize the multiplications (using associativity) to get

$$\bar{x} = 2 \times \frac{9}{300} + 3 \times \frac{13}{300} + 4 \times \frac{22}{300} + 5 \times \frac{33}{300} + \cdots + 10 \times \frac{29}{300} + 11 \times \frac{10}{300} + 12 \times \frac{9}{300}$$

In this new form, we compute the mean by first multiplying each outcome by its proportion in the distribution and then summing those results. In this case, the mean is $\bar{x} = 7.09$.

What's the point of all this? Well, for a (discrete) random variable, we don't have a list of values or a list of counts. We only have a probability for each outcome. But that is enough to compute a mean using the reorganized arithemetic. That is, the mean of a discrete random variable is computed by first multiplying each outcome by the probability for that outcome and then adding up those results.