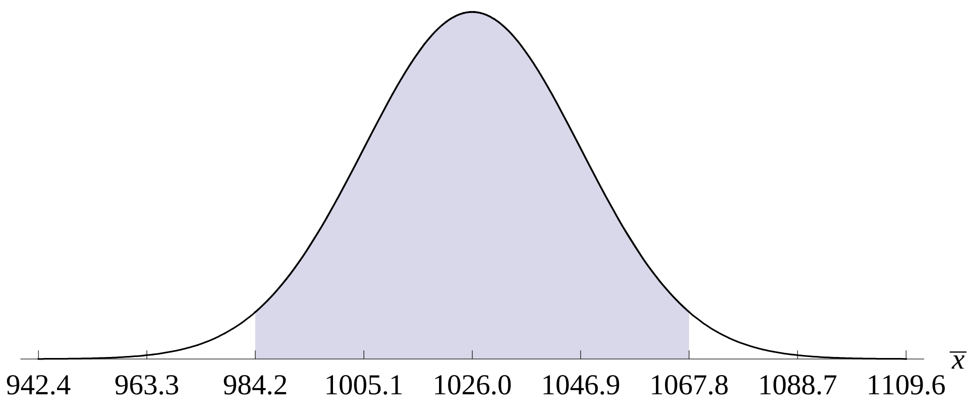
Confidence intervals

- consider SAT scores (math+verbal) with normal distribution having mean $\mu = 1026$ and standard deviation $\sigma = 209$
- for SRSs of size $n = 100$, sample means $\bar{x}$ have normal distribution with

$$\begin{aligned} \text{mean} \qquad & \mu_{\bar{x}} = \mu = 1026 \\ \text{standard deviation} \qquad & \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 20.9 \end{aligned}$$

- 95% of all sample means $\bar{x}$ fall within $2\sigma_{\bar{x}} = 2(20.9) = 41.8$ of $\mu_{\bar{x}} = 1026$



- since $\mu_{\bar{x}} = \mu = 1026$, we can say that 95% of all sample means $\bar{x}$ fall within $2\sigma_{\bar{x}} = 41.8$ of the population mean μ
- so, μ is within $2\sigma_{\bar{x}} = 41.8$ of 95% of all possible values of $\bar{x}$
- now think about all of the intervals of the form $[\bar{x} - 2\sigma_{\bar{x}}, \bar{x} + 2\sigma_{\bar{x}}] = [\bar{x} - 41.8, \bar{x} + 41.8]$ or $\bar{x} \pm 41.8$
- μ must fall into 95% of these intervals
- these intervals are called 95% *confidence intervals*
- in practice,
  - select *one* SRS
  - measure to get *one* sample mean
  - construct *one* confidence interval
  - report that confidence interval

- want information on value of a mean μ for a population
- select a SRS of size n and measure sample mean $\bar{x}$
- suppose sample means are normally distributed with mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$
  - 95% of all sample means $\bar{x}$ fall within $2\sigma_{\bar{x}}$ of $\mu_{\bar{x}}$
  - since $\mu_{\bar{x}} = \mu$, we can say that 95% of all sample means $\bar{x}$ fall within $2\sigma_{\bar{x}}$ of the population mean μ
  - so, μ is within $2\sigma_{\bar{x}}$ of 95% of all possible values of $\bar{x}$
- now think about all of the intervals of the form $[\bar{x} - 2\sigma_{\bar{x}}, \bar{x} + 2\sigma_{\bar{x}}]$
  - μ must fall into 95% of these intervals
  - these intervals are called 95% *confidence intervals*
- for any one SRS with $\bar{x}$, it is tempting to say that there is probability 95% that the interval $[\bar{x} - 2\sigma_{\bar{x}}, \bar{x} + 2\sigma_{\bar{x}}]$ contains μ
- but, for any one SRS with $\bar{x}$, the 95% confidence interval $[\bar{x} - 2\sigma_{\bar{x}}, \bar{x} + 2\sigma_{\bar{x}}]$ either contains μ or does not contain μ so no probability statement applies
- a probability statement does apply to the *random process* of selecting a SRS, measuring to get $\bar{x}$, and then constructing the interval $[\bar{x} - 2\sigma_{\bar{x}}, \bar{x} + 2\sigma_{\bar{x}}]$
- so, we can say that a 95% confidence interval $[\bar{x} - 2\sigma_{\bar{x}}, \bar{x} + 2\sigma_{\bar{x}}]$ is produced by a process that has probability 95% of producing an interval containing the true population mean μ

- constructing a a confidence interval for a population mean (when the population standard deviation is known)
  - choose a *confidence level* C

    Example: $C = 95\% = 0.95$

  - determine the z-score $z^\star$ for that confidence level

    Example: for $C = 95\% = 0.95$, have $z^\star = 2$

  - choose a sample size n

    Example: $n = 100$

  - compute the standard deviation $\sigma_{\bar{x}}$

    Example: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{209}{\sqrt{100}} = 20.9$

  - compute the *margin of error* $m = z^\star \sigma_{\bar{x}}$

    Example: $m = z^\star \sigma_{\bar{x}} = 2(20.9) = 41.8$

  - measure to get a specific value for $\bar{x}$

    Example: $\bar{x} = 1050$

  - report the confidence interval $\bar{x} \pm m$

    Example: $\bar{x} \pm m = 1050 \pm 41.8$

    or

    $$[\bar{x} - m, \bar{x} + m] = [1050 - 41.8, 1050 + 41.8] = [1008.2, 1091.8]$$

- refinement:
  - value of $z^\star = 2$ for $C = 0.95$ comes from 68-95-99.7 rule
  - that rule is a convenient approximation
  - more precise value for $C = 0.95$ is $z^\star = 1.96$

    Example: $m = z^\star \sigma_{\bar{x}} = (1.96)(20.9) = 40.96$