A connection between significance tests and confidence intervals

- consider a two-sided significance test with hypotheses

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

- what difference between the sample mean $\bar{x}$ and the hypothesized population mean μ_0 puts us at the borderline between rejecting the null hypothesis and not rejecting it at the $\alpha = 0.05$ significance level?
- let $z^\star$ be the z-score for this borderline situation so $z^\star = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}$
- so, the borderline difference is $\bar{x} - \mu_0 = z^\star \sigma_{\bar{x}}$
- need to find $z^\star$ so that

$$P\text{-value} = P\Big(Z < -z^\star \text{ or } Z > z^\star\Big) = 0.05$$

- from previous experience, know $z^\star$ is about 2; more precisely, can use $z^\star = 1.96$
- this is the same value for $z^\star$ we use to construct the 95% confidence interval with margin of error $z^\star \sigma_{\bar{x}}$
- therefore, doing a two-sided significance test at the $\alpha = 0.05$ significance level is the same thing as computing a 95% confidence interval and then checking if μ_0 is in that interval
- to summarize
  - μ_0 outside the 95% CI is the same as rejecting the null hypothesis at the 0.05 significance level
  - μ_0 in the 95% CI is the same as not rejecting the null hypothesis at the 0.05 significance level
- more generally, doing a two-sided significance test at the significance level α is the same thing as computing a confidence interval with confidence level $C = 1 - \alpha$ and then checking if μ_0 is in that interval