Name ______________________________

MATH 302 Partial Differential Equations Fall 2007 Exam #1

Instructions:

Do your own work. You may consult your class notes and the course text. Do not consult other sources. Do not discuss generalities or specifics of the exam with anyone except me.

Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work (both computations and reasoning).

Each of the four problems has equal weight.

The exam is due Tuesday, October 10. For at least one problem, you will need to submit a *Mathematica* notebook. For this, send me an e-mail with an attachment. Name the file you send `Math302Exam1_XX.nb` where XX are your initials.

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1. Consider the function f defined on $[-4, 4]$ by $f(x) = x^2 - x$.

   (a) Compute the complex form of the Fourier series for f.

   (b) Use your result from (a) to express this Fourier series in terms of cosines and sines.

2. For f defined on $[-1, 1]$ by $f(x) = x^3$, the Fourier series is

$$\text{FS}(x) = \frac{2}{\pi^3} \sum_{n=1}^{\infty} (-1)^n \left(\frac{6}{n^3} - \frac{\pi^2}{n} \right) \sin\big(n\pi x\big).$$

   (a) Find the smallest value of N so that the partial sum $S_N(x)$ approximates f with a mean square error less than 0.01.

   (b) Make one plot showing the graphs of $S_N(x)$ and $f(x)$.

   (c) Find the maximum pointwise difference between $S_N(x)$ and $f(x)$ for the interval $(-1, 1)$ and compare this with 0.01. Comment on this comparison.

3. (a) Let f be a function defined on the interval $[a, b]$. Find an infinite series representation of f in terms of trigonometric functions of period $b - a$. Give formulas for the coefficients in this infinite series representation.
   Hint: Use a linear transformation to map the interval $[a, b]$ to an interval of the form $[-p, p]$ and then use what you know about Fourier series representations for $[-p, p]$.

   (b) Use your result from (a) to compute a series representation for the function $f(x) = e^x$ on $[-1, 3]$. Make a plot of the series representation for the interval $[-5, 8]$.

4. Consider the initial boundary value problem for the wave equation given by

$$\begin{aligned}
\frac{\partial^2 u}{\partial t^2}(x,t) &= c^2 \frac{\partial^2 u}{\partial x^2}(x,t) && 0 < x < L, \quad t > 0 \\
u(0,t) &= 0 && t > 0 \\
u(L,t) &= 0 && t > 0 \\
u(x,0) &= f(x) && 0 < x < L \\
\frac{\partial u}{\partial t}(x,0) &= g(x) && 0 < x < L
\end{aligned}$$

Choose

$$f(x) = \begin{cases} 2x & \text{if } 0 < x \le \frac{1}{2} \\ 1 & \text{if } \frac{1}{2} < x < \frac{3}{4} \\ 4 - 4x & \text{if } \frac{3}{4} \le x < 1 \end{cases}$$

For each of the following choices of $g(x)$, find the solution and make an animation showing one full cycle (in t) of the solution.

(a) $g(x) = 0$

(b) $g(x) = \begin{cases} x & \text{if } 0 < x \le \frac{1}{2} \\ 1 - x & \text{if } \frac{1}{2} < x < 1 \end{cases}$