Name ______________________________

MATH 301 Differential Equations Spring 2006 Exam #2

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

You can use integration aids such as a table of integrals.

---

1. (a) Describe the structure of the general solution for a homogeneous n^{th} order linear differential equation. (5 points)

   (b) Describe the structure of the general solution for a nonhomogeneous n^{th} order linear differential equation. (5 points)

2. Suppose that λ is a repeated real root for the characteristic equation of $ay'' + by' + cy = 0$ where a, b, and c are constants. Show that $\{e^{\lambda t}, te^{\lambda t}\}$ is a basis for the set of solutions (i.e, that this is a fundamental set of solutions). (8 points)

3. Find the largest interval for which the existence-uniqueness theorem guarantees a unique solution to the following initial-value problem: (8 points)

$$(\cos t)\, y''' + y'' + t\, y = \frac{1}{t-2}, \quad y(1) = 4, \quad y'(1) = 7, \quad y''(1) = -2.$$

4. For each of the following, solve the given differential equation or initial-value problem. (12 points each)

   (a) $y'' - 6y' + 34y = 0, \quad y(0) = 0, \quad y'(0) = 20$

   (b) $y'' + 3y' - 10y = 4e^{3t}$

   (c) $y'' + 3y' - 10y = 4\sin(3t)$

   (d) $t^2y'' + 4ty' + 2y = 0$ for $t > 0$
   Hint: Look for solutions of the form t^λ where λ is a constant.

5. An object of mass m hangs on a vertical spring of stiffness k. An external force is applied with the form $F_{\text{ext}} = \alpha t$ with α a constant. Assume the object is in the equilbrium position at rest at $t = 0$. Also assume that damping is negligible.

   (a) Set up an initial-value problem to model this situation. (8 points)

   (b) Solve the initial-value problem you set up. (12 points)

   (c) Under what conditions, if any, on the parameters k, m, and α will the object be in the equilibrium position for some positive value of t? (6 points)