# The damped spring

We've modeled a damped spring-mass system using

$$m\,y''(t) + \gamma\,y'(t) + k\,y(t) = 0.$$

We have expressed solutions in terms of

$$\alpha = \frac{\gamma}{2m} \qquad \text{and} \qquad \omega_0 = \sqrt{\frac{k}{m}}.$$

Consider a series of experimental runs in which $k$, $m$ and the initial conditions are held constant while $\gamma$ is varied from run to run. For each run, a value of $\gamma$ is fixed while data on $y$ versus $t$ is collected and plotted. Think of each plot as a frame in a movie with frames ordered by increasing values of $\gamma$. (That is, think of $\gamma$ as the playing time for the movie.)

We know that the form of the solution for our model depends on the value of $\gamma$ relative to $\gamma_c = 2\sqrt{km}$. If we use solutions to make movie frames showing $y$ versus $t$, what will we see as the movie plays through the "time" $\gamma_c$? Will the solutions for $\gamma < \gamma_c$ flow continuously through the solution to $\gamma_c$ to solutions to $\gamma > \gamma_c$ or will there be a discontinuous jump at $\gamma_c$?

1. For the case $\gamma < \gamma_c$, find the specific solution with the initial conditions $y(0) = 0$ and $y'(0) = v_0$.
2. For the case $\gamma = \gamma_c$, find the specific solution with the initial conditions $y(0) = 0$ and $y'(0) = v_0$.
3. For the case $\gamma > \gamma_c$, find the specific solution with the initial conditions $y(0) = 0$ and $y'(0) = v_0$.
4. Analyze the limit of your solution in 1 as $\gamma \to \gamma_c^-$. Compare this limit with your solution from 2.
5. Analyze the limit of your solution in 3 as $\gamma \to \gamma_c^+$. Compare this limit with your solution from 2.
6. Will the movie be continuous or discontinuous at $\gamma_c$? Explain how you reach your conclusion.