MATH 280A   Multivariate Calculus   Fall 2006   Project #3

Instructions

   You should submit a carefully written report addressing the problems given below. You are encouraged to discuss ideas with others for this project. If you do work with others, you must still write your report independently.

   Use the writing conventions given in *Some notes on writing in mathematics*. You should include enough detail so that a reader can follow your reasoning and reconstruct your work. You should not show every algebraic or arithmetic step. All graphs should be done carefully on graph paper or using appropriate technology.

   For this project, you can do either one of the two problems. You may not use any double integral or iterated integral feature of computing technology.

   The project is due on Thursday, November 30.

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1. **Evaluating double integrals**

   For each of the following, evaluate the given double integral. Give an exact value if you can. Otherwise, give the best approximation you can.

   (a) $\displaystyle\iint_D e^{-x^2-y^2}\,dA$ where D is the disk of radius 1 centered at the origin

   (b) $\displaystyle\iint_D e^{-x^2-y^2}\,dA$ where D is the square of side length 2 centered at the origin

   (c) $\displaystyle\iint_D e^{-xy}\,dA$ where D is the square of side length 2 centered at the origin

2. **Electron probabilities**

   The $n = 3$, $l = 2$, $m = 0$ state of a free hydrogen atom has an electron probability density (per volume) given by

   $$p(\rho, \phi, \theta) = \frac{1}{39366\pi}\rho^4 e^{-2\rho/3}(3\cos^2\phi - 1)^2$$

   where (ρ, ϕ, θ) are spherical coordinates as we use them in class. The radial coordinate ρ is measured in units of *Bohr radii* where the Bohr radius is equal to about 5.3×10^{-11} meters. (So, for example, $\rho = 2$ means a radial distance of 2 Bohr radii.)

   (a) Compute an expression for the probability of finding the electron between $\rho = a$ and $\rho = b$ for a hydrogen atom in this state.

   (b) Compute the probability of finding the electron in each of the unit intervals $\rho = n$ to $\rho = n + 1$ for n between 0 and 25. Identify the interval in which the electron is most likely to be found.

   (c) Compute the probability of finding the electron anywhere in space. Does this result make sense?