Name ______________________________

MATH 280A | Multivariate Calculus | Fall 2006 | Project #1

Instructions

You should submit a carefully written report addressing the problems given below. You are encouraged to discuss ideas with others for this project. If you do work with others, you must still write your report independently.

Use the writing conventions given in *Some notes on writing in mathematics.* You should include enough detail so that a reader can follow your reasoning and reconstruct your work. You should not show every algebraic or arithmetic step. All graphs should be done carefully on graph paper or using appropriate technology.

For this project, Problem 1 is worth 80% of the maximum credit while Problems 2 and 3 are each worth 10% of the maximum credit. Problem 4 is a bonus for fun. You don't need to submit Problem 4.

The project is due in class on Friday, October 6.

Parametric descriptions of surfaces

1. **Background:** A line in the plane can be described by a linear equation in two variables of the form $Ax + By + C = 0$. Alternatively, a line in the plane can be described parametrically by a vector formula of the form $\vec{R}(t) = \vec{R}_0 + t\vec{d}$ where $\vec{R}_0$ is the position vector for a point on the line, $\vec{d}$ is a direction vector parallel to the line, and t is a parameter. The line is the set of all points with position vectors $\vec{R}(t)$ as t ranges over all real numbers.

   A plane in space can be described by a linear equation in three variables of the form $Ax + By + Cz + D = 0$.

   (a) Come up with a way to describe a plane in space by a parametric vector formula. Include a geometric description of the vectors you use in the parametric description. Hint: Think about using two parameters.

   (b) Use your result for the first part to give a parametric description of the plane that goes through the points $(3, 2, 1)$, $(-6, 5, 9)$ and $(2, 1, -4)$.

2. Come up with a way to parametrize a right circular cylinder.

3. Consider the parametric equation

   $$\vec{R}(u, v) = \sin u \cos v\, \hat{\imath} + \sin u \sin v\, \hat{\jmath} + \cos u\, \hat{k}$$

   where the parameter u has values in the interval $[0, \pi]$ and the parameter v has values in the interval $[0, 2\pi]$.

   (a) Use words and pictures to describe the surface traced out by this parametric equation.

   (b) Consider fixing a value $u = u_0$ and letting v vary through the values in $[0, 2\pi]$. The vector $\vec{R}(u_0, v)$ traces out a curve on the surface. Describe and/or draw this type of curve.

   Now consider fixing a value $v = v_0$ and letting u vary through the values in $[0, \pi]$. The vector $\vec{R}(u, v_0)$ traces out a curve on the surface. Describe and/or draw this type of curve.

   (c) Consider the point on the surface given by $u_0 = \pi/4$ and $v_0 = \pi/6$. Compute a vector normal (i.e., perpendicular) to the surface by taking the cross product of two vectors, one tangent to the curve of constant u through the point and the other tangent to the curve of constant v through the point. Draw a figure showing the surface and the point. Draw the two tangent vectors and the normal vector at the point.

4. Consider the parametric equation

$$\vec{R}(u,v) = (3+\cos u)\cos v\,\hat{\imath} + (3+\cos u)\sin v\,\hat{\jmath} + \sin u\,\hat{k}$$

where the parameter u has values in the interval $[0, 2\pi]$ and the parameter v also has values in the interval $[0, 2\pi]$.

(a) Use words and pictures to describe the surface traced out by this parametric equation.

(b) Consider fixing a value $u = u_0$ and letting v vary through the values in $[0, 2\pi]$. The vector $\vec{R}(u_0, v)$ traces out a curve on the surface. Describe and/or draw this type of curve.

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