

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

For problems requiring a conclusion about convergence or divergence, provide an argument or evidence to support your conclusion.

There is a total of 140 points possible.

1. Briefly explain the difference between the *definite integral* $\int_a^b f(x) dx$ and the *indefinite integral* $\int f(x) dx$. (5 points)
2. What is the difference between a *sequence* and a *series*? (5 points)
3. *For this problem, you may not use your calculator.* Evaluate $\int x^2 \sin(5x^3) dx$. (10 points)
4. *For this problem, you may not use your calculator.* Evaluate $\int x^2 e^{3x} dx$. (10 points)
5. *For this problem, you may not use your calculator.* Evaluate $\int_3^8 \left(x^2 + 4 + \frac{1}{x^2}\right) dx$.
You can stop computing when only arithmetic remains. (10 points)
6. Consider the function $A(x) = \int_0^x e^{-s^2} ds$.
 - (a) What is $A'(x)$? (5 points)
 - (b) Sketch a graph of $A(x)$. (5 points)
7. A cake is removed from an oven and left to cool. The temperature T of the cake is given as a function of time t by $T(t) = 70 + 280e^{-1.4t}$ where T is measured in $^{\circ}F$ and t is measured in hours. Compute the average temperature of the cake during the time interval from $t = 0$ to $t = 2$ hours. (10 points)
8. *For this problem, you may not use built-in or programmed integration features on your calculator.* Approximate $\int_0^1 \sin(x^2) dx$ to within $\pm 10^{-2}$. (10 points)

There is more on the flip side.

9. Consider the region bounded between the curves $y = 32 - x^4$ and $y = 4x^2$. Set up an integral or integrals to compute the volume of the solid generated by revolving this region around the x -axis. *You do not need to evaluate the integral or integrals.* (10 points)
10. Set up an integral or integrals to compute the surface area of a cone of height H and base radius R . *You do not need to evaluate the integral or integrals.* (10 points)
11. You win the lottery and are given the following two options for receiving your winnings:
- A. You and your heirs forever receive income continuously at a rate of $\frac{10}{t+1}$ in dollars per year.
 - B. You and your heirs forever receive income continuously at a rate of $\frac{100}{(t+1)^2}$ in dollars per year.
- If your goal is to maximize the total winnings for you and your heirs, which option do you pick? Justify your response. (10 points)
12. *For this problem, you may not use built-in or programmed summation features on your calculator.* Compute the sum of $\sum_{k=1}^{\infty} \frac{7}{5^{2k}}$. (10 points)
13. Determine if the series $\sum_k \frac{k}{\sqrt{k^5 + 4}}$ converges or diverges. Give an argument to support your claim. (10 points)
14. Determine if the series $\sum_k \frac{(2k)!}{3^k k!(k+1)!}$ converges or diverges. Give an argument to support your claim. (10 points)
15. Find the Taylor series based at $x = 0$ for the function $f(x) = \sin(x^3)$. (10 points)