

**Instructions:** You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. Set up an integral equal to the line integral for the vector field  $\vec{F} = y\hat{i} - x\hat{j}$  and the curve  $x^2 + y = 9$  from  $(1, 8)$  to  $(3, 0)$ . The integral should be expressed entirely in terms of one variable (of your choice). You do not need to evaluate the integral. (15 points)
  
2. Set up an iterated integral in two variables equal to the surface integral for the vector field  $\vec{F} = 5\hat{i} + z\hat{j} + x\hat{k}$  and the surface consisting of the piece of the cylinder  $y = z^2$  bounded by the planes  $x = 0$ ,  $x = 2$ , and  $y = 9$ . The iterated integral should be expressed entirely in terms of two variables (of your choice). You do not need to evaluate the integral. (15 points)
  
3. The accompanying figure shows a vector field  $\vec{F}$ , an oriented curve  $C$ , and points  $P$  and  $Q$ .
  - (a) Determine if the value of  $\int_C \vec{F} \cdot d\vec{R}$  is positive or negative. Explain how you arrive at your conclusion. (5 points)
  - (b) Determine if the divergence of  $\vec{F}$  at  $P$  is positive or negative. Explain how you arrive at your conclusion. (5 points)
  - (c) Determine if the  $\hat{k}$ -component of the curl of  $\vec{F}$  at  $Q$  is positive or negative. Explain how you arrive at your conclusion. (5 points)
  
4. Consider the vector field  $\vec{F} = \sin y\hat{i} + 2xz\hat{j} + 4z^2\hat{k}$ .
  - (a) Compute the divergence of  $\vec{F}$ . (5 points)
  - (b) Compute the curl of  $\vec{F}$ . (5 points)
  - (c) Is  $\vec{F}$  a conservative vector field? Explain how you arrive at your conclusion. (4 points)
  
5. Prove that the divergence of the curl of a vector field is zero. That is, prove the identity  $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$ . (8 points)

6. Compute the line integral of the vector field  $\vec{F} = (2y^2 + 5)\hat{i} + 4xy\hat{j}$  for a curve  $C$  that starts at the point  $(2, -1)$  and ends at  $(0, 4)$ . (15 points)
7. Suppose  $\vec{\nabla} \times \vec{F} = \vec{0}$  at all points on a surface  $S$  in space. Use Stokes' Theorem to show that  $\oint_C \vec{F} \cdot d\vec{R} = 0$  where  $C$  is the curve that forms the edge of the surface  $S$ . (8 points)
8. Consider the vector field  $\vec{F} = (z - x)\hat{i} + (x - y)\hat{j} + (y - z)\hat{k}$ . Use the Divergence Theorem to evaluate  $\oiint_S \vec{F} \cdot d\vec{A}$  where  $S$  is the sphere of radius 4 centered at the origin.  
Hint: You should be able to do this without having to work hard to evaluate an integral. Recall that  $\iiint_D dV$  is equal to the volume of the solid region  $D$ . (10 points)

### Conclusions of the Fundamental Theorems

FT of Calculus:  $\int_a^b F'(x) dx = F(b) - F(a)$

FT for Line Integrals:  $\int_C \vec{\nabla}V \cdot d\vec{R} = V(Q) - V(P)$

Stokes' Theorem:  $\iint_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{A} = \oint_C \vec{F} \cdot d\vec{R}$

Green's Theorem:  $\iint_D \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dA = \oint_C (u\hat{i} + v\hat{j}) \cdot d\vec{R}$

Divergence Theorem:  $\iiint_S (\vec{\nabla} \cdot \vec{F}) dV = \oiint_S \vec{F} \cdot d\vec{A}$