Name ____________________________

MATH 221B | Multivariate Calculus | Spring 2004 | Exam #4

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. Evaluate the double integral of the function $f(x, y) = xy^2$ for the region of the xy-plane in the first quadrant bounded by the x-axis and the curves $y = x^2$ and $y = 2 - x^2$. (20 points)

2. Set up, but do not evaluate, an iterated integral (or integrals) that gives the area of the planar region inside one "petal" of the polar curve $r = 3\sin(2\theta)$. Express your result entirely in terms of a single coordinate system (of your choice). (20 points)

3. A solid sphere of radius R is sliced into two pieces by a plane with the distance of a between the center of the sphere and the plane (with $a < R$). Set up, but do not evaluate, an iterated integral (or integrals) for the volume of the smaller piece. Express your result entirely in terms of a single coordinate system (of your choice). (20 points)

4. Set up, but do not evaluate, an iterated integral (or integrals) that gives the total mass of the tetrahedron in the first octant bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $z = 12 - 3x - 4y$ with a mass density equal to the square of the distance from the xz-plane. Express your result entirely in terms of a single coordinate system (of your choice). (20 points)

5. (a) Give a definition of the double integral $\iint_D f\,dA$ for the function f on the planar region D. (10 points)

   (b) Give a geometric argument for why the correct volume element in cylindrical coordinates is $dV = r\,dr\,d\theta\,dz$. Include at least one relevant figure. (10 points)