Name ____________________

MATH 221B Multivariate Calculus Spring 2004 Exam #3

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. In a given coordinate system for a particular room, the temperature is given by $T = f(x, y, z)$. A fly moves through the room with position given by $\vec{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath} + z(t)\,\hat{k}$. For units, assume temperature is measured in °C, lengths are measured in meters, and time is measured in seconds.
   (a) Write out the relevant chain rule for computing $\dfrac{dT}{dt}$ in this situation. (6 points)
   (b) Suppose $f(x, y, z) = x^2 + y^2 z + xz$ and $\vec{r}(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath} + t\,\hat{k}$. Compute the rate of change in temperature with respect to time experienced by the fly at time $t = \pi$. Include units in your answer. (8 points)
2. Consider the function $f(x, y, z) = xyz + 3x + y^2$.
   (a) Compute the gradient of f. (4 points)
   (b) Give the value and direction (as a unit vector) of the greatest rate of change in f at the point $(1, -1, 4)$. (8 points)
   (c) Compute the rate of change in f at the point $(1, -1, 4)$ in the direction of the point $(2, 3, 1)$. (6 points)
3. The accompanying plot shows the gradient vector field for a function $f : \mathbb{R}^2 \to \mathbb{R}$ for inputs (x, y) in $[-10, 10] \times [-10, 10]$.
   (a) Sketch the level curve that passes through the point $(8, 5)$. (4 points)
   (b) Sketch the path of steepest ascent starting at the point $(2, 6)$. (4 points)
   (c) Estimate the input (x, y) for which the graph of f is steepest. (2 points)
   (d) Estimate the coordinates of the input (x, y) for each local minimum, local maximum, and saddle in the plot region. (6 points)
4. Consider the function $f(x, y) = x \sin(x + y)$.
   (a) Show that $(0, \pi)$ is a critical point for f. (9 points)
   (b) Determine if f has a local minimum, a local maximum, or a saddle at $(0, \pi)$. (9 points)
5. Find the global minimum and global maximum for $f(x, y) = (x - 3)^2 - (y - 1)^2 + x + y$ on the rectangle with vertices at $(0, 0)$, $(4, 0)$, $(4, 2)$, and $(0, 2)$. (18 points)
6. Consider a (right circular) cylinder of radius r and height h. Use the method of Langrange multipliers to find the dimensions that give the maximum volume for a fixed surface area A_0 (including top and bottom). Note that the volume is given by $V = \pi r^2 h$ and the surface area is given by $A = 2\pi rh + 2\pi r^2$. (16 points)