Name ________________________________

MATH 221B Multivariate Calculus Spring 2004 Exam #2

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. For each of the following, state a definition equivalent to that given in the text.
   (a) *the derivative of a vector-output function* $\vec{F}(t)$ (6 points)
   (b) *the partial derivative with respect to* y *of the function* $f(x,y,z)$ (6 points)

2. A wire is wrapped around a circular cylinder of radius 4 cm and length 20 cm so that there are 10 complete wraps of the wire. Give a vector-output function that parametrizes the shape of this wire. Include a domain for the function. (12 points)

3. Compute the derivative of the vector-output function $\vec{F}(t) = e^{3t}\,\hat{\imath} + 2t^3\,\hat{\jmath} + t\cos t\,\hat{k}$. (9 points)

4. Consider the output curve for the vector-output function $\vec{F}(t) = (t^2-3)\hat{\imath}+(t+1)\hat{\jmath}+(6t-7)\hat{k}$. and the plane with equation $4x + 2y - z = 5$.
   (a) Show that the output curve intersects the plane for $t = 2$. (5 points)
   (b) Find the acute angle between a normal to the plane and the curve at the point of intersection. (7 points)

5. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^3+y^3}$ does not exist. (10 points)

6. Compute all first and second partial derivatives of the function $f(x,y) = x^2\sin(3xy^2)$. (15 points)

7. Compute all first partial derivatives of the function $G(p,q,r) = pqe^{pr^2}$. (9 points)

8. Find the equation of the tangent plane for the function $f(x,y) = 4x^2y^3 - 7xy^2$ for $(x,y) = (1,2)$. (12 points)

9. For an ideal gas, the temperature T is related to the pressure P and volume V by $T = \alpha PV$ where α is a constant. The quantity $\frac{\Delta T}{T}$ represents a percentage change in the temperature. Show that this percentage change $\frac{\Delta T}{T}$ is *approximately* equal to the sum of the percentage changes in the pressure and volume. (9 points)