Name ______________________

MATH 221A | Multivariate Calculus | Spring 2004 | Exam #1

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. Consider the vector $\vec{u} = 2.1\hat{\imath} + 4.3\hat{\jmath} - 9.2\hat{k}$.

   (a) Compute the magnitude of $\vec{u}$. (4 points)

   (b) Find a unit vector in the direction of $\vec{u}$. (4 points)

2. Consider a coordinate system for the plane with unit coordinate vectors $\hat{\imath}$ and $\hat{\jmath}$. Suppose the vector $\vec{v}$ has components $\vec{v} = 2\hat{\imath} - 7\hat{\jmath}$ in this coordinate system. Now consider a second coordinate system given by rotating the original axes 90° clockwise. Let $\hat{I}$ and $\hat{J}$ be the unit coordinate vectors for this new system. Express $\vec{v}$ in components in this second coordinate system. (6 points)

3. Consider a pyramid with square base of side length 6 and a vertex 4 units above the center of the base. Use vectors to find the angle between a side of the base and an edge from a corner of the base to the vertex. (8 points)

4. (a) Give the geometric definition of dot product. (6 points)

   (b) Give the geometric definition of cross product. (6 points)

5. For each of the following, determine if the given expression is a scalar, a vector, or undefined. (2 points each)

   (a) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$

   (b) $(\vec{a} \times \vec{b}) \times (\vec{c} \cdot \vec{d})$

   (c) $[(\vec{a} \cdot \vec{b})\vec{c}] \cdot \vec{d}$

   (d) $[(\vec{a} \times \vec{b}) \times \vec{c}] \times \vec{d}$

6. Show that $(3\vec{u} + 5\vec{v}) \times (2\vec{u} - \vec{v})$ is parallel to $\vec{u} \times \vec{v}$. (6 points)

7. Consider a plane that contains the point Q and is perpendicular to the vector $\vec{n}$.

   (a) Explain how we know the point P is on the plane if $\overrightarrow{QP} \cdot \vec{n} = 0$. You can use words and pictures for this. (4 points)

   (b) Show how to go from the vector equation $\overrightarrow{QP} \cdot \vec{n} = 0$ to a coordinate equation of the form $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$. (4 points)

   (c) Find the equation of the plane that contains the points $Q(a, 0, 0)$, $R(0, b, 0)$, and $S(0, 0, c)$. (6 points)

8. Consider a line that contains the point Q and is parallel to the vector $\vec{d}$.

   (a) Explain how we know the point P is on the line if there is a scalar t such that $\overrightarrow{QP} = t\vec{d}$. You can use words and pictures for this. (4 points)

   (b) Show how to go from the vector equation $\overrightarrow{QP} = t\vec{d}$ to the parametric equations for the line. (4 points)

   (c) Find the parametric equations of the line that contains the point $Q(2, -4, 9)$ and is perpendicular to the plane $7x - 5y + 2z = 14$. (6 points)

9. Consider the plane $6x - 8y + z = 21$ and the line with parametric equations $x = 25 - 2t$, $y = 4 + 2t$, and $z = -9 + 4t$.

   (a) Find the point of intersection between the plane and the line. (5 points)

   (b) Find the acute angle between a normal vector for the plane and a direction vector for the line. (5 points)

10. Consider the surface given by the equation $\dfrac{x^2}{4} - \dfrac{y^2}{25} + z^2 = 1$.

   (a) Sketch cross-sections for the three coordinate planes. Give scales on each plot. (9 points)

   (b) Sketch and/or describe the surface. (5 points)