Name ______________________________

MATH 301 Differential Equations Spring 2005 Exam #4

Instructions:

Do your own work. You may consult class notes, the course text, or other books. Give a reference if you use some source other than class notes or the course text. You may not consult or discuss the exam with other people.

Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work (both computations and reasoning). If you are not confident in some result, you will receive more credit if you make a note of this and comment on where you might be going wrong or on alternate approaches you might try.

You can use technology as needed. When you do, give some indication of what you use and how you use it.

The exam is due Wednesday, May at 4:00 pm.

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1. Find the general solution for the system

$$\begin{aligned}\frac{dx_1}{dt} &= x_1 - 4x_2 - x_3\\ \frac{dx_2}{dt} &= 3x_1 + 2x_2 + 3x_3\\ \frac{dx_3}{dt} &= x_1 + x_2 + 3x_3\end{aligned}$$

2. Find the specific solution for the initial value problem

$$\begin{aligned}\frac{dx_1}{dt} &= -3x_1 - 36x_2\\ \frac{dx_2}{dt} &= x_1 + 9x_2\end{aligned}$$

with $x_1(0) = 1$ and $x_2(0) = -4$.

3. Analyze the phase portrait of the following system of equations. Make specific claims and conjectures about features in the phase portrait. You can use both analytic and numerical methods (with tools such as the java applet *JOdeApplet2D*). Claims that you can prove analytically will have more value than conjectures based on numerical evidence alone. Include plots (possibly printed from *JOdeApplet2D*) to illustrate yours conjectures and claims.

$$\begin{aligned}\frac{dx}{dt} &= 2x + y^3\\ \frac{dy}{dt} &= x + 2y\end{aligned}$$

4. Consider a mass-spring system with two objects and three springs as shown in the figures below. The springs are identical, each having a natural length of L and a spring constant k. The system is set up in a gap of length $3L$ so that none of the springs is stretched or compressed in the equilibrium position shown in the figure on the left. Let $x_1(t)$ be the position of the left mass with respect to its equilibrium position and let $x_2(t)$ be the position of the right mass with respect to its equilibrium position as shown in the figure on the right. The objects are also identical, each with a mass of m. Assume friction can be ignored.

   (a) Give some argument to justify the following equations as a model for this physical situation.

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\begin{aligned}
m\frac{d^2x_1}{dt^2} &= -kx_1 + k(x_2 - x_1) \\
m\frac{d^2x_2}{dt^2} &= -kx_2 - k(x_2 - x_1)
\end{aligned}
$$

   (b) Rewrite this system of two second-order equations as a system of four first-order equations.

   (c) Find the general solution of the first-order system. You might find it convenient to introduce a new parameter defined by $\omega^2 = k/m$.

   (d) Your general solution should be a linear combination of four solutions. Describe the physical motion that corresponds to each of the four solutions. For each, include graphs of x_1 vs. t and x_2 vs. t and a description of the motion using words and/or pictures.