

Instructions:

Do your own work. You may consult class notes, the course text, or other books. Give a reference if you use some source other than class notes or the course text.

Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work (both computations and reasoning). If you are not confident in some result, you will receive more credit if you make a note of this and comment on where you might be going wrong or on alternate approaches you might try.

For any solution in the form of a power series, determine a recurrence equation for the coefficients. Solve the recurrence equation or compute at least 4 nonzero terms. If the series has a finite number of nonzero terms, compute all coefficients.

Do any four of the six problems. Circle the problem number for each problem you submit. Each problem has a maximum value of 25 points.

The exam is due Wednesday, April 13 at 4:00 pm.

1. Find the general solution of $x''(t) - 2x'(t) + x(t) = \frac{e^t}{t}$ for $t > 0$

2. Find the general solution of $t^2 \frac{d^2x}{dt^2} - 2t \frac{dx}{dt} + 2x = t^m$ for $t > 0$.

3. Find the general solution of $x'' + t^3x' + tx = 0$.

4. Find the general solution of $2t^2x'' + (7t^2 + 7t)x' - 3x = 0$ for $t > 0$.

5. Find the general solution of the third-order equation $\frac{d^3x}{dt^3} + t \frac{dx}{dt} + x = 0$.

6. Find the specific solution of the initial-value problem

$$x''(t) + (t+1)^2x'(t) - 4(t+1)x(t) = 0, \quad x(-1) = 0, \quad x'(-1) = 1.$$