Name ______________________

MATH 301 Differential Equations Spring 2005 Exam #2

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

Each problem has a value of 20 points.

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1. Find the specific solution of the initial value problem

$$(D-2)^2(D+3)[x(t)] = 0, \qquad x(0)=4, \quad x'(0)=-1, \quad x''(0)=1.$$

2. Find the general solution of the differential equation $\dfrac{d^2x}{dt^2} - 2\dfrac{dx}{dt} - 15x = 6e^{5t}$.

3. The position $x(t)$ of an object on a spring subject to a damping force is modeled by

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0.$$

For a particular set-up, suppose $m = 0.1$ kg, $b = 0.2$ N/(m/s) and $k = 1.7$ N/m. Find the specific solution for the initial condition $x(0) = 0.2$ m and $x'(0) = 0$. Sketch a plot of position vs. time for this solution. Briefly describe the motion of the object in words.

4. Show that

$$x(t) = c_1\frac{1}{t^2} + c_2t^3 + t^3\ln t$$

is the general solution of the differential equation

$$t^2\frac{d^2x}{dt^2} - 6x = 5t^3$$

for the interval $I = (0, \infty)$.

5. Do one of the following two problems. If you do work on both, clearly indicate which one you want to have evaluated.

   (a) Consider a constant-coefficient linear ODE in the form $P(D)[x(t)] = E(t)$. In the method of undetermined coefficients, one looks for a particular solution in the form of the general solution to the homogeneous ODE $A(D)P(D)[p(t)] = 0$ where $A(D)$ annihilates $E(t)$. In your own words, outline the procedure for the method of undetermined coefficients and explain why this method works (i.e., why it results in a particular solution of $P(D)[x(t)] = E(t)$).

   (b) Let $h_1(t)$ and $h_2(t)$ be solutions of a linear, homogeneous, second order ODE $L[x] = 0$ which is normal on an interval I. Prove that if $h_1(t)$ and $h_2(t)$ are both zero for some t_0 in I, then $\{h_1(t), h_2(t)\}$ is linearly dependent.