Name _____

MATH 301 Differential Equations Spring 2005 Exam #1

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. For each of the following, solve the given differential equation or initial-value problem. (20 points each)

   (a) $\dfrac{dx}{dt} = -x\sin t + \sin t$

   (b) $\dfrac{dx}{dt} = -\dfrac{x\sin(tx)}{t\sin(tx) + 2x}, \qquad x(\pi) = 1$

   (c) $\dfrac{dx}{dt} = t^4x^2, \qquad x(-1) = 3$

   (d) $\dfrac{dx}{dt} = \dfrac{2x^4 + t^4}{tx^3}$

2. Consider a species with a natural per capita rate of change of 0.03 per year. Assume that in addition to the natural rate of change, the population is harvested (think of fishing) at a constant rate of 1500 individuals per year.

   (a) Let $x(t)$ denote the population at time t. Briefly explain how the differential equation

   $$\frac{dx}{dt} = 0.03x - 1500 \qquad x(0) = x_0.$$

   matches the assumptions made in the model. (5 points)

   (b) Sketch a slope field for $0 \le t$ and $0 \le x$ with enough detail to show all interesting features. (5 points)

   (c) On your slope field, sketch enough solution curves to show all the possible types of behavior as t increases. For each type of behavior, give the relevant range of initial values x_0 and briefly describe the behavior. (5 points)

   (d) One of the behaviors in (c) should be that the population reaches 0 in finite time. Find an expression for this extinction time T in terms of the intial population x_0. (5 points)