Name ______________________________

MATH 122 Calculus and Analytic Geometry II Fall 2005 Project #2

Instructions

You should submit a carefully written report addressing the problems given below. You are encouraged to discuss ideas with others for this project. If you do work with others, you must still write your report independently.

Use the writing conventions given in *Some notes on writing in mathematics.* You should include enough detail so that a reader can follow your reasoning and reconstruct your work. You should not show every algebraic or arithmetic step. All graphs should be done carefully on graph paper or using appropriate technology.

For this project, the first problem is worth 80% of the maximum credit, the second problem is worth 15% of the maximum credit, and presentation is worth 5% of the maximum credit.

The project is due in class on Monday, October 10.

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1. For each type of numerical approximation (rectangular, trapezoidal, Simpson's rule), we have a way to compute an upper bound on the error. We can use this to get an approximation that is guaranteed to have an error less than a desired tolerance.

   In practice, the following quick "rule of thumb" is often used: Start by computing an approximation A_1. Double the number of subintervals to 2 and compute A_2. If A_1 and A_2 differ by less than the desired tolerance, stop and use A_2. If the difference is bigger than the desired tolerance, double the number of subintervals again. Repeat until the difference is less than the desired tolerance. Use the last approximation you compute. This approximation often has an error less than the desired tolerance but there is no guarantee.

   Use this idea to approximate $\int_0^1 e^{-x^2}\,dx$ with an error (probably) less than 0.001.

2. Here's a variation that often saves computational time. Compute A_1 and A_2 as before. If the difference is less than the tolerance, stop and use A_2. If the difference is bigger than the tolerance, treat each of the two subintervals separately. Allot half of the original tolerance to each subinterval. For each subinterval, compute approximations using one and two "subsubintervals" and compare the difference to the allotted tolerance. If the difference is less than the allotted tolerance, stop. If not, treat each of the "subsubintervals" separately, dividing the allotted tolerance in half again. Repeat until the allotted tolerance is achieved for each piece.

   Use this variation to approximate $\int_0^1 e^{-x^2}\,dx$ with an error (probably) less than 0.001.