Approximating functions by polynomials

1. (a) Find the linear function that best approximates $f(x) = e^x$ at $x = 0$.
   Note: A linear function is also known as a polynomial of degree 1. Denote this linear function by $p_1(x)$ where the p stands for polynomial and the subscript 1 gives the degree.

   (b) Find the quadratic function (i.e., polynomial of degree 2) that best approximates $f(x) = e^x$ at $x = 0$.

   (c) Find the cubic function (i.e., polynomial of degree 3) that best approximates $f(x) = e^x$ at $x = 0$.

   (d) Make a conjecture about the quartic function (i.e., polynomial of degree 4) that best approximates $f(x) = e^x$ at $x = 0$.

   (e) Make a conjecture about the polynomial of degree n that best approximates $f(x) = e^x$ at $x = 0$.

2. Repeat the steps in Problem 1 for $f(x) = \dfrac{1}{1 - x}$ at $x = 0$.

3. Repeat the steps in Problem 1 for $f(x) = \sin x$ at $x = 0$.

4. Repeat the steps in Problem 1 for $f(x) = \cos x$ at $x = 0$.

5. Repeat the steps in Problem 1 for $f(x) = \ln x$ at $x = 1$.