Constructing definite integrals

1. Consider the problem of computing the total mass of a column of air. The density of air decreases as height above sea level increases. Let h be a height above sea level measured in meters. Let $\rho(h)$ be the density of air, measured in kg/m^3, at height h.

   (a) Construct a definite integral to compute the total mass of air in a cylindrical column of radius R and height H with its base at sea level.

   (b) Compute the total mass of air if $\rho(h) = \rho_0 e^{-kh}$ where ρ_0 and k are positive constants.

   (c) Get a numerical value for the total mass using the values $\rho_0 = 1.22$ kg/m^3, $k = 1.1 \times 10^{-4}$ m^{-1}, $R = 1$ m and $H = 10000$ m.

2. Consider the problem of computing the total number of bacteria in a circular petri dish. The bacteria colony is more dense at the center than at the edges of the petri dish. Let r denote a radial distance from the center of the dish measured in centimeters. Let $\sigma(r)$ be the density of the bacteria colony, measured in number per square centimeter, at radius r.

   (a) Construct a definite integral to compute the total number of bacteria in a petri dish of radius R.

   (b) Compute the total number of bacteria if the density is σ_0 at the center of the dish and decreases linearly to zero at the edge of the dish.

   (c) Get a numerical value for the total number with the density as in (b) and the values $\sigma_0 = 5.4 \times 10^3$ per cm^2 and $R = 5.5$ cm.

3. Here is a fact about continuously compounded interest: An amount A (in dollars) in an account earning interest at a continuously compounded rate r (in % per year) has a value after τ years of $Ae^{r\tau}$.

   Consider the problem of computing the future value of deposits in an investment account. Money is deposited into the account at a known rate and the account earns interest compounded continuously. Let t be a time in years and $d(t)$ be the deposit rate (in dollars per year) at time t.

   (a) Construct a definite integral to compute the value of an account T years in the future.

   (b) Compute the future value if the deposit rate is a constant d_0 in dollars per year.

   (c) Get a numerical value for the value after 5 years with a constant deposit rate of \$1000 per year and an interest rate of 6%.