Population models

1. Consider a population that has a constant per capita rate of change in its natural state. Suppose humans harvest this population at a constant rate. (As an example, you might think of fishing regulations that allow a fixed number of fish to be caught each year.)
   (a) Set up a differential equation for the population.
   (b) Sketch a slope field for this differential equation.
   (c) On your slope field, sketch some solution curves for different initial values.
   (d) Organize solutions into classes with common features. For each class, describe the common features and give a characterizing condition in terms of the parameters in the model.
   (e) Solve the differential equation. Find the specific solution with an initial condition $p(0) = p_0$.
   (f) For one group of solutions, you should find that the population goes to zero in finite time. Find an expression for this time T in terms of the other parameters in the model.

2. Consider a population model in which the per capita rate of change is proportional to the difference between a constant K and the population. (The constant K is usually called the *carrying capacity* of the environment.)
   (a) Set up a differential equation for the population.
   (b) Sketch a slope field for this differential equation.
   (c) On your slope field, sketch some solution curves for different initial values.
   (d) Organize solutions into classes with common features. For each class, describe the common features and give a characterizing condition in terms of the parameters in the model.
   (e) Solve the differential equation. Find the specific solution with an initial condition $p(0) = p_0$.