MATH 121A   Calculus and Analytic Geometry I   Spring 2004   Final Exam

Part A Instructions: Do the work for these problems on separate paper. Clearly box or circle the final answer for each computational problem. Do reasonable simplification. Each problem has a value of 3 points. Each problem will be evaluated as correct or not correct with no partial credit.

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1. State the definition of derivative.

2. State the First Fundamental Theorem of Calculus. Include the hypotheses and the conclusion.

3. Evaluate the limit $\lim\limits_{x \to 0} \dfrac{\sin x}{x}$.

4. Evaluate the limit $\lim\limits_{x \to 3} \dfrac{x - 3}{x^2 - 9}$.

5. Compute the derivative of $f(x) = 3x^4 - 7x^3 + 2x^2$.

6. Compute the derivative of $f(x) = x \sin x$.

7. Compute the derivative of $f(t) = \cos(t^2)$.

8. Compute the derivative of $g(x) = (x + 2)^4 (x + 5)^9$.

9. Compute the derivative of $h(u) = \sqrt{u^3 + 7u}$.

10. Compute the derivative of $f(x) = \dfrac{x}{2x + 5}$.

11. Compute the derivative of $f(x) = \ln(x^3)$.

12. Compute the derivative of $g(\theta) = \tan \theta \sec \theta$.

13. Compute the derivative of $f(x) = e^{ax}$.

14. Compute the second derivative of $f(x) = xe^x$.

15. Find the slope of the line tangent to the graph of function $f(x) = 3x^2 + 4x$ for $x = 5$.

16. The position of a car on a straight road is given by $s(t) = 4 - t^3$ with s measured in miles and t measured in hours. Find the velocity of the car at $t = 2$.

17. Find and classify (as local minimum, local maximum, or neither) all critical points of the function $f(x) = x^4 + 4x^3$.

18. Evaluate the indefinite integral $\int 5x^3 \, dx$.

19. Find all antiderivatives of the function $f(x) = x + \sin x$.

20. Evaluate the definite integral $\int_2^4 x^2 \, dx$.

21. Evaluate the definite integral $\int_0^{\pi/4} \sec^2 x \, dx$.

22. Evaluate the definite integral $\int_2^3 (x^2 + e^x) \, dx$.

23. Evaluate the definite integral $\int_{-1}^2 (5x^3 + 6x^2) \, dx$.

24. Find the derivative with respect to x of $\int_{-4}^x e^{u^4} \, du$.

Part B Instructions: Do any three of the following five problems. Do the work for these problems on separate paper. On this page, circle each of the problem numbers for the three problems you are submitting. Each problem is worth 10 points. I will assign partial credit for incomplete or incorrect work.

I. Find the slope $\frac{dy}{dx}$ of the line tangent to the curve with equation $x^3y - xy^2 = 24$ at the point $(x, y) = (3, 1)$.

II. Consider the function $f(x) = xe^{-ax}$ where a is a positive constant. Use calculus techniques to plot a graph of f that shows all of the essential features. On the plot, label the essential features (such as local minima, local maxima, and inflection points).

III. The radius r of a sphere is measured to within $\pm\Delta r$. The volume is calculated using $V = \frac{4}{3}\pi r^3$. Find an approximate relation between the percentage error in the radius r and the percentage error in the volume V.

IV. A lighthouse shines a horizontal beam of light that rotates at a constant rate of 3 revolutions per minute. The lighthouse is on an island located 400 meters off a straight coastline. For miles in both directions, the coast is a vertical cliff. In part of each revolution, the beam of light makes a spot of light that moves along the cliff face. How fast is this spot of light moving along the cliff face when the spot passes through the point on the cliff face closest to the lighthouse?

V. A window is to be made in the shape of a rectangle capped by a semicircle as shown in the figure. The area is to be a total of 20 square feet. Find the dimensions that minimize the perimeter of the window. Note: Do not count the dashed line in the figure as part of the perimeter.