MATH 121A   Calculus and Analytic Geometry I   Spring 2004   Exam #4

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

   This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. (a) State the Mean Value Theorem.   (10 points)

   (b) Draw and label a figure that illustrates a geometric interpretation of the Mean Value Theorem.   (6 points)

2. Analyze each of the following limits.

   (a) $\lim\limits_{x\to 0} \dfrac{x\cos(x) - x}{x^3}$ (14 points)   (b) $\lim\limits_{x\to+\infty} \left(\dfrac{x-12}{x}\right)^{3x}$ (12 points)

3. Plot the graph of a function that has all of the following features: (24 points)

   (i) $f(-1) = 0$, $f(1) = -3$, $f(3) = 0$, $f(5) = 2$
   (ii) a vertical asymptote at $x = -2$
   (iii) $\lim\limits_{x\to-\infty} f(x) = 2$ and $\lim\limits_{x\to+\infty} f(x) = -\infty$
   (iv) $f'(x) > 0$ for $x < -2$ and $1 < x < 5$
   (v) $f'(x) < 0$ for $-2 < x < 1$ and $5 < x$
   (vi) $f''(x) > 0$ for $x < -2$ and $-2 < x < 3$
   (vii) $f''(x) < 0$ for $3 < x$

4. Consider the function $f(x) = \dfrac{4x^3}{x^3 - 1}$. Use calculus techniques for each of the following.

   (a) Find and analyze any vertical asymptotes.   (4 points)

   (b) Analyze any horizontal asymptotes.   (4 points)

   (c) Find all intervals of inputs x for which the function is positive and the all intervals of inputs x for which the function is negative.   (6 points)

   (d) Find all intervals of inputs x for which the function is increasing and all intervals of inputs x for which the function is decreasing.   (6 points)

   (e) Find all intervals of inputs x for which the function is concave up and all intervals of inputs x for which the function is concave down.   (6 points)

   (f) Sketch a graph of the function and label any asymptotes, zeros, local minima, local maxima, and inflection points.   (8 points)