Name ______________________________

MATH 121B   Calculus and Analytic Geometry I   Spring 2004   Exam #3

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. (a) Give a definition of *a local (or relative) minimum of a function* f. (4 points)

   (b) Give a definition of *a critical point of a function* f. (4 points)

   (c) State the Extreme Value Theorem. (4 points)

2. Consider the relation $z \sin x + xz^3 = 8\pi$.

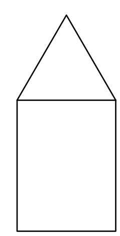
   (a) Compute the derivative $\dfrac{dz}{dx}$. (10 points)

   (b) Compute the slope of the tangent line for the graph of this relation at the point $(\pi, 2)$. (4 points)

3. A lighthouse shines a horizontal beam of light that rotates at a constant rate of 3 revolutions per minute. The lighthouse is on an island located 400 meters off a straight coastline. For miles in both directions, the coast is a vertical cliff. In part of each revolution, the beam of light makes a spot of light that moves along the cliff face. How fast is this spot of light moving along the cliff face when the spot passes through the point on the cliff face closest to the lighthouse? (20 points)

4. You are designing a window in the shape of a rectangle surmounted by an equilateral triangle as shown in the figure. You want to frame the window with wood trim, including the edge between the rectangle and the triangle. You have 30 feet of trim available. What dimensions should you use to get the largest possible area for the window? (20 points)

   Note: The area of an equilateral triangle with sides of length s is given by the formula $A_\triangle = \dfrac{\sqrt{3}}{4}s^2$.



5. Consider the function $f(x) = (x^2 - 3)e^{-x}$.

   (a) Use calculus techniques to find and classify (as local minimizer, local maximizer, or neither) all critical points for the function f (12 points)

   (b) Find the global minimum and global maximum for the function f on the interval $[-2, 5]$. (8 points)

   Note: Remember to use the chain rule in computing the derivative of e^{-x}.

6. The area A of a square is to be measured with a measurement error of ΔA. The length L of the square is to be computed with the formula $L = \sqrt{A}$. Find an approximate relation between the percentage error in the area and the percentage error in the length. (14 points)