

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

1. (a) Give a definition of *the derivative of the function f at x* (5 points)
 (b) State the quotient rule for derivatives. (5 points)

2. Use the definition of derivative to show that $\frac{d}{dx}[x^2] = 2x$. (8 points)

3. Compute the first, second, and third derivatives of the function $f(x) = 4x^3 + \frac{2}{x}$. (9 points)

4. For each of the following, compute the derivative of the given function. Do some reasonable simplification for each result. (6 points each)

(a) $f(x) = 15x^4 - 7x^2 + 2x - 9$

(c) $f(x) = \ln(\sin(x))$

(b) $g(t) = \left(\frac{6t-2}{8t^3+3t^2}\right)^3$

(d) $h(x) = \sqrt{e^x + x}$

(e) $f(x) = axe^{bx}$

5. Find the equation of the tangent line for the function $f(x) = \frac{6x}{x^2+1}$ for $x = 1$. (10 points)

6. Find all values of x for which the graph of $f(x) = 5x^3 - 7ax$ has a horizontal tangent line. (8 points)

7. On a particular straight line, the temperature T (in °C) is measured as a function of position s (in feet) and found to be given by the formula

$$T(s) = 0.003s^2 + 0.04s.$$

An object moves on this same line with position s (in feet) given as a function of time t (in seconds) by the formula

$$s(t) = 17 - 4t^3.$$

- (a) Find the rate of change in temperature with respect to position for $s = -15$ feet. Include correct units in the result. (5 points)
- (b) Find the rate of change in position with respect to time for $t = 2$ seconds. Include correct units in the result. (5 points)
- (c) Find the rate of change in temperature with respect to time for $t = 2$ seconds. Include correct units in the result. (5 points)

8. The plot below shows the graph of a function f . Plot a graph of the derivative f' on the axis provided below. As part of this, choose a scale for the vertical axis and label the axis accordingly. (10 points)

