

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. Give a definition, equivalent to that in the text, for each of the following terms. (6 points each)

(a) *the function f has the limit L at a* (b) *the function f is continuous at a*

2. For each of the following, solve the given equation or inequality. (6 points each)

(a) $x^2 - 5x - 6 \leq 0$

(b) $\log(5x - 1) = \log(6x) - \log(2)$

3. Simplify $\tan(\sin^{-1} x)$. That is, rewrite $\tan(\sin^{-1} x)$ as an algebraic expression. (6 points)

4. Plot at least one full cycle of the graph of $y = 3 \cos(4(x - \pi))$. Include scales on each axis. Give the period of this function. (6 points)

5. (a) The table to the right gives input/output pairs for a function f . Use this information to plot four points on the graph of the inverse function f^{-1} . (4 points)

x	$f(x)$
0	10
1	6
2	5
3	1

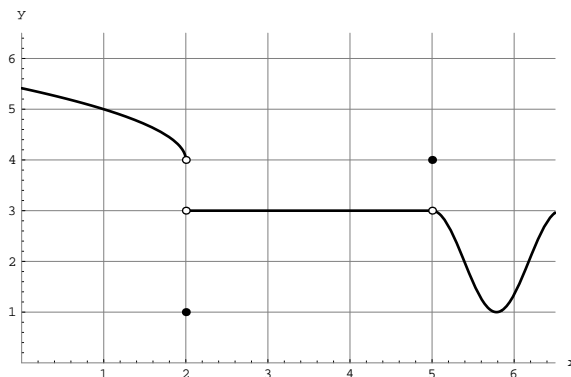
- (b) The table to the right gives input/output pairs for a function g . Explain why this function does *not* have an inverse function. (4 points)

x	$g(x)$
0	4
1	0
2	4
3	7

6. Suppose a chunk of ice is melting so that the weight W of ice (in pounds) at time t (in minutes) is given by $W(t) = W_0 e^{-kt}$. At time $t = 0$, the chunk of ice weighs 6 pounds and 15 minutes later, the ice weighs 5.2 pounds. Find the time at which the ice will weigh 4 pounds. (8 points)

7. Analyze $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$ using a table of input/output pairs. (8 points)

8. The plot below shows the graph of a function f . Use the graph to analyze continuity of the function for the interval $[0, 6]$. Determine the points at which the function is discontinuous. For each point of discontinuity, state specifically how the definition of continuity fails to hold. (8 points)



9. For each of the following, evaluate the limit using techniques that give an exact result if possible. Show enough details to make your methods clear to a reader. (6 points each)

(a) $\lim_{x \rightarrow 100} \frac{\sqrt{x} - 10}{x - 100}$

(c) $\lim_{a \rightarrow b} \frac{a - b}{a^2 - b^2}$

(b) $\lim_{x \rightarrow 1} \frac{\cos(\pi x)}{\cos(2\pi x)}$

(d) $\lim_{x \rightarrow -3} f(x)$ where $f(x) = \begin{cases} x^2 & \text{if } x < -3, \\ 9 & \text{if } x = -3 \\ 4x + 10 & \text{if } x > -3. \end{cases}$

10. Consider two functions g and h . Suppose we know these facts

- g is continuous for all real numbers
- h is continuous for all real numbers
- $h(7) = 0$
- $\lim_{x \rightarrow 7} \frac{g(x)}{h(x)} = 4$.

- (a) What must be true about $g(7)$? Explain how you reach your conclusion. (4 points)

- (b) In the table below, fill in values for $g(x)$ that are consistent with the facts you are given. (4 points)

x	$h(x)$	$g(x)$
6.9	0.2035	
6.99	0.0201	
6.999	0.0020	
6.9999	0.0002	