

Some parallels in the stories of derivative and integral

	Derivative	Integral
Original motivation	Slope of tangent line for graph of f	Area of region under graph of f
Definition	$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$	$\int_a^b f(x) dx = \lim_{\Delta x \rightarrow 0} \sum_{k=1}^n f(x_k^*) \Delta x.$
Interpretations	<ol style="list-style-type: none"> 1. Slope of tangent line for graph of f 2. Rate of change in one quantity with respect to a second 	<ol style="list-style-type: none"> 1. Area of region under graph of f 2. Accumulation of one quantity with respect to a second
Rules/tools for computing	<ol style="list-style-type: none"> 1. Derivatives of basic functions (power, trig, exp, log) 2. Rules for combinations of functions (sum, product, quotient, chain) 	<ol style="list-style-type: none"> 1. Fundamental Theorem of Calculus 2. Rules for combinations of functions

In first introducing the idea of derivative, we started by thinking about the problem of computing slope of a tangent line. The definition of derivative is motivated by the idea of finding a general way to compute the slope of a tangent line for the graph of a function. In the end, the definition of derivative can stand alone without any interpretation as slope (or rate of change).

A similar story happens for introducing integrals. We start by thinking about the problem of calculating area of a region. The definition of integral is motivated by the idea of finding a general way to compute the area of a region between the graph of a function and the horizontal axis. In the end, the definition of integral can stand alone without any interpretation as area (or accumulation).