Name _____

MATH 302 Partial Differential Equations Fall 2002 Final Exam

Instructions: Do your own work. You may consult class notes, the course text, or other books. Give a reference if you use some source other than class notes or the course text. Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work. If you are not confident in some result, you will receive more credit if you make a note of this and comment on where you might be going wrong or on alternate approaches you might try. The exam is due Friday, December 20 at 2:00 pm.

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1. Solve the following initial-value problem for the heat equation with a source term on the real line:

$$\begin{aligned} \frac{\partial u}{\partial t} &= c^2 \frac{\partial^2 u}{\partial x^2} + h(x) && -\infty < x < \infty, \quad t > 0 \\ u(x,0) &= f(x) && -\infty < x < \infty. \end{aligned}$$

2. Solve the following boundary-value problem for Laplace's equation on an infinite strip:

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= 0 && -\infty < x < \infty, \quad 0 < y < L \\ u(x,0) &= f(x) && -\infty < x < \infty \\ u(x,L) &= g(x) && -\infty < x < \infty. \end{aligned}$$

3. (a) Describe the separation of variables technique for solving a boundary-value or initial boundary-value problem for a second-order partial differential equation in two variables. Be as general as possible. Include a discussion on conditions necessary for separation of variables to work and refer to relevant aspects of Sturm-Liouville theory.

   (b) Consider the general linear second-order partial differential equation in two variables:

$$a(x,y)\frac{\partial^2 u}{\partial x^2} + b(x,y)\frac{\partial^2 u}{\partial x \partial y} + c(x,y)\frac{\partial^2 u}{\partial y^2} + d(x,y)\frac{\partial u}{\partial x} + e(x,y)\frac{\partial u}{\partial y} + f(x,y)u = 0.$$

   Find conditions on the coefficient functions a, b, c, d, e, and f so that the PDE is separable. Look for the most general conditions possible.