# Additional problems for Section 3.1

1. Compute the value of $\int_0^1 x^2\,dx$ using the definition of definite integral.

   Hint: $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

2. Let $f$ and $g$ be functions for which we know the following:

   $$\int_3^5 f(x)\,dx = 7, \qquad \int_5^7 f(x)\,dx = 4, \qquad \text{and} \qquad \int_3^5 g(x)\,dx = -2 \qquad .$$

   Find a value for each of the following.

   (a) $\int_5^3 f(x)\,dx$

   (b) $\int_3^7 f(x)\,dx$

   (c) $\int_3^5 4f(x)\,dx$

   (d) $\int_3^5 \Big(2f(x) + 5g(x)\Big)\,dx$

3. Consider the function $f$ defined on the interval $[0,1]$ by

   $$f(x) = \begin{cases} 0 & \text{for } x = 0 \\ \frac{1}{x} & \text{for } 0 < x \leq 1. \end{cases}$$

   Argue that this function is not integrable by considering limits of Riemann sums.

4. Consider the function $f$ defined on the interval $[0,1]$ by

   $$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$$

   Argue that this function is not integrable by considering limits of Riemann sums.